Prova scritta del 27. 03. 2008 (fila 1)

C1) $X \sim N(\mu, \sigma^2 = 4)$ $\quad \mu: P[X > 3.6] = 0.0778$

$Z = \frac{X-\mu}{\sigma} \sim N(0,1)$

$$P\left[Z > \frac{3.6-\mu}{2}\right] = P[Z > z_\alpha] = 0.0778$$

$$P[Z \leq z_\alpha] = 1 - 0.0778 = 0.9222 \Rightarrow z_\alpha = 1.42$$

$$\Rightarrow \frac{3.6-\mu}{2} = 1.42 \Rightarrow \mu = 0.76$$

C2) $p = 0.0002 = 2 \cdot 10^{-4}$ $\quad P(X \geq 2)$ in 20 anni

in 20 anni ci sono $52 \cdot 20 = 1040$ settimane $\Rightarrow m = 1040$

$X \sim B(m, p)$ ma $mp = 1040 \cdot 2 \cdot 10^{-4} = 0.208 <<$

$\Rightarrow$ B può essere approssimata da $\mathcal{P}(\lambda = mp)$

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad x = 0, 1, \ldots$$

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X=0] - P[X=1]$$
$$= 1 - e^{-\lambda} - \lambda e^{-\lambda}$$
$$= 1 - e^{-0.208}(0.208 + 1) = 0.01885$$

si può ragionare anche sui giorni

$\bar{m} = 20 \cdot 365 = 7300$ $\quad 7300 : 7 \cong 1043$ settimane

e in questo caso $mp = 0.2086$

C3) $$P[X > \sqrt{e} \mid -1 < X < e^3] = \frac{P[X > \sqrt{e} \cap -1 < X < e^3]}{P[-1 < X < e^3]}$$

ma $-1 < \sqrt{e} < e^3$

$$= \frac{P[\sqrt{e} < X < e^3]}{P[-1 < X < e^3]}$$

$$= \frac{F(e^3) - F(\sqrt{e})}{F(e^3) - F(-1)}$$

$$= \frac{1 - \ln\sqrt{e}}{1 - 0} = 1 - \frac{1}{2}\ln e = \frac{1}{2}$$

C4) $T \sim Unif(15°C, 20°C) \Rightarrow f_T = \begin{cases} \frac{1}{5} & \text{se } 15<T<20 \\ 0 & \text{altrove} \end{cases}$

$P_1 = P[X > 18°C] = \frac{1}{5}(20-18) = \frac{2}{5}$

$P[X \leq 18°C] = \frac{3}{5}$

se in 4gg su 7 $T > 18°C$ in 3gg su 7 $T \leq 18°C$

$Y \sim B(n=7, p=\frac{3}{5})$

$P[Y=3] = \binom{7}{3}\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^4 = \frac{7!}{3!\,4!}\,\frac{3^3}{5^3}\cdot\frac{2^4}{5^4} = \frac{7\cdot 3^2\cdot 2^4}{5^6}$

QT)

A indip da B $\rightarrow P(A\cap B) = P(A)\cdot P(B)$

A indip da C $\rightarrow P(A\cap C) = P(A)\cdot P(C)$

B, C incomp $\rightarrow P(B\cap C) = 0$

$$P[A|B\cup C] = \frac{P[A\cap(B\cup C)]}{P(B\cup C)} = \frac{P[(A\cap B)\cup(A\cap C)]}{P(B)+P(C)-\underbrace{P(B\cap C)}_{0}}$$

$$= \frac{P[A\cap B] + P[A\cap C] - \overset{=0}{P}[A\cap \overbrace{B\cap C}^{=\phi}]}{P[B]+P[C]}$$

$$= \frac{P[A]\cdot P[B] + P[A]\cdot P[C]}{P[B]+P[C]} = P[A]$$

E1)

$\sum f_{X,Y} = 1 \Rightarrow P/2 + P/4 + \frac{1}{4} + \frac{1}{2} = 1 \Rightarrow \frac{3}{4}p + \frac{3}{4} = 1 \Rightarrow p = \frac{1}{3}$

$\rho(5X, 3Y) = \rho_{X,Y}$ per le proprietà di ρ.

$$\rho_{X,Y} = \frac{cov[X,Y]}{\sigma_X \cdot \sigma_Y}$$

$$E[XY] = -1\cdot 3 \cdot \frac{1}{2} + 1 \cdot 1 \cdot \frac{1}{4} = -\frac{5}{4}$$

$$E[X] = 1 \cdot \frac{1}{4} + 3 \cdot \frac{1}{2} = \frac{7}{4}$$

$$E[Y] = -1 \cdot \left(\frac{1}{6} + \frac{1}{2}\right) + 1 \cdot \left(\frac{1}{12} + \frac{1}{4}\right) = -\frac{2}{3} + \frac{1}{3} = -\frac{1}{3}$$

$$\Rightarrow \operatorname{cov}[X,Y] = -\frac{5}{4} + \frac{7}{4}\cdot\frac{1}{3} = \frac{7-15}{12} = -\frac{2}{3}$$

$$E[X^2] = 1 \cdot \frac{1}{4} + 9 \cdot \frac{1}{2} = \frac{19}{4}$$

$$E[Y^2] = 1 \cdot \frac{2}{3} + 1 \cdot \frac{1}{3} = 1$$

$$\Rightarrow \sigma_X^2 = \frac{19}{4} - \frac{49}{16} = \frac{27}{16}$$

$$\sigma_Y^2 = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\Rightarrow \underline{\rho} = \frac{-2/3}{(3\sqrt{3}/4)\cdot(2\sqrt{2}/3)} = -\frac{2}{3} \cdot \frac{2}{\sqrt{6}} = \underline{-\frac{2}{9}\sqrt{6}}$$

E2)

$M_1' = \mu_1'$ dove $M_1' = \bar{X}_n$ e $\mu'_1 = E[X]$ da calcolare

$$E[X] = \int_2^{+\infty} x \cdot \frac{3}{\theta}\, e^{-\frac{3}{\theta}(x-2)}\, dx$$

$x-2 = y \qquad x = y+2 \qquad dx = dy$

$x = 2 \to y = 0$

$x = +\infty \to y = +\infty$

$$E[X] = \int_0^{+\infty} (y+2)\, \frac{3}{\theta}\, e^{-\frac{3}{\theta}y}\, dy$$

$$= \int_0^{\infty} \left(-\frac{3}{\theta}\right)(-y)\, e^{-\frac{3}{\theta}y}\, dy + (-)2 \int_0^{\infty} (-)\frac{3}{\theta}\, e^{-\frac{3}{\theta}y}\, dy$$

$$= \left|(-y)\, e^{-\frac{3}{\theta}y}\right|_0^{+\infty} + \overset{(-\theta/3)}{\underset{(-\frac{3}{\theta})}{\int_0^{\infty}}} e^{-\frac{3}{\theta}y}\, dy - 2 \left| e^{-\frac{3}{\theta}y}\right|_0^{\infty}$$

$$= \lim_{y \to +\infty} \frac{-y}{e^{\frac{3}{\theta}y}} - \frac{\theta}{3}\left[e^{-\frac{3}{\theta}y}\right]_0^{\infty} - 2\left[e^{-\frac{3}{\theta}y}\right]_0^{\infty}$$

$$\overset{H}{=} \lim_{y \to +\infty} \underbrace{\frac{-1}{\frac{3}{\theta}e^{\frac{3}{\theta}y}}}_{\downarrow\, 0} + \frac{\theta}{3} + 2 = \frac{\theta}{3} + 2$$

Perciò $\bar{X}_m = \frac{\theta}{3} + 2 \Rightarrow \hat{\theta} = 3\bar{X}_m - 6$

$$E[\hat{\theta}] = E[3\bar{X}_m - 6] = 3E[\bar{X}_m] - 6 = 3\mu - 6 = 3\left(\frac{\theta}{3} + 2\right) - 6 \equiv \theta$$

$\hat{\theta}$ è corretto

$$MSE[T] = var[T] + \overset{0}{\cancel{D^2(T)}}$$

$$var[3\bar{X}_m - 6] = var[3\bar{X}_m] = 9\, var[\bar{X}_m] = 9\frac{\sigma^2}{m}$$

$$\sigma^2 = var[X] = E[X^2] - E[X]^2$$

$$E[X^2] = \int_2^{+\infty} x^2 \cdot \frac{3}{\theta} e^{-\frac{3}{\theta}(x-2)} dx = \int_0^{\infty} (y+2)^2 \frac{3}{\theta} e^{-\frac{3}{\theta}y} dy$$

$$= \cdots = 2\frac{\theta^2}{9} + \frac{4}{3}\theta + 4$$

$$\Rightarrow \sigma^2 = 2\frac{\theta^2}{9} + \frac{4}{3}\theta + 4 - \left(\frac{\theta}{3} + 2\right)^2 = \frac{\theta^2}{9}$$

$$\Rightarrow MSE[\hat{\theta}] = \frac{\theta^2}{m}$$

$\lim_{m \to \infty} MSE(\hat{\theta}) = 0 \Rightarrow \hat{\theta}$ è consistente

Prova scritta del 03.09.2008 (fila 1)

C1) $X \sim N(\mu = 6.3, \sigma^2 = 4)$

$$P[\sqrt[3]{X} \geq 2] = P[X \geq 2^3] = P\left[Z > \frac{8-6.3}{2}\right] = P[Z > 0.85]$$

$$= 1 - P[Z \leq 0.85] \underset{\substack{\uparrow \\ \text{TAB}}}{=} 1 - 0.80234 = 0.19766$$

C2) 3R, 4B

$X \sim Geom(p)$ $\quad f_X = p\, q^{x-1}$ dove $q = 1-p$

$P[X=3] = p(1-p)^2$ $\quad$ p è da calcolare

$$p = \frac{\binom{R}{1}\binom{B}{1}}{\binom{R+B}{2}} = \frac{\binom{3}{1}\cdot\binom{4}{1}}{\binom{7}{2}} = \frac{12}{\frac{7!}{2!\,5!}} = \frac{4}{7}$$

$$\Rightarrow P[X=3] = \frac{4}{7}\cdot\frac{9}{49} = \frac{36}{343}$$

C3) $X \sim \exp(\lambda)$ $\quad E[X] = 400$

poiché $E[X] = \frac{1}{\lambda}$ $\Rightarrow$ $\lambda = \frac{1}{400}$

$$P[X \geq 100 \mid X = 500] \underset{\substack{\uparrow \\ \text{mancanza memoria}}}{=} P[X \geq 100] = 1 - P[X < 100]$$

$$= 1 - F(100) = 1 - \left[1 - e^{-\frac{1}{400}\cdot 100}\right]$$

$$= e^{-1/4} = \frac{1}{\sqrt[4]{e}} = 0.7788$$

Ricorda che se $f_X = \lambda e^{-\lambda x} \rightarrow F(x) = 1 - e^{-\lambda x}$

C4)

$X \sim$ Poisson $= P(\lambda)$ $\quad f_X = \frac{\lambda^x}{x!} e^{-\lambda}$

$\alpha = 0.03$

$\lambda = 5\alpha = 5 \cdot 0{,}03 = 0.15$

$P[X=0] = e^{-0.15} = 0.86071$

QT) X, Y indip $\Rightarrow E[XY] = E[X] \cdot E[Y] \Rightarrow cov[X,Y] = 0$

$cov[X-2Y, X+3] = cov[X,X] + \underbrace{cov[X,3]}_{=0} + cov[-2Y,X]$
$\qquad - \underbrace{cov[-2Y,3]}_{=0}$

$= cov[X,X] - 2\underbrace{cov[Y,X]}_{=0} = var[X]$

E1)

$M_1' = \mu_1' \quad M_1' = \bar{X}_m \quad \mu_1' = \mu = E[X]$

$E[X] = \int_0^{+\infty} x \cdot \frac{16}{\theta^2} \cdot x \cdot e^{-\frac{4}{\theta}x} dx = \left(+\frac{4}{\theta}\right) \int_0^{\infty} x^2 \left(\frac{4}{\theta} e^{-\frac{4}{\theta}x}\right) dx$

$= \frac{4}{\theta} \int_0^{\infty} y^2 \left(\frac{4}{\theta} e^{-\frac{4}{\theta}y}\right) dy \quad$ posto $Y \sim exp\left(\lambda = \frac{4}{\theta}\right)$

$= \frac{4}{\theta} E[Y^2]$

ma $E[Y^2] = var[Y] + E[Y]^2$

e se $Y \sim exp$ $\quad E[Y] = \frac{1}{\lambda}$, $\quad var[Y] = \frac{1}{\lambda^2}$

quindi $E[Y^2] = \frac{2}{\lambda^2} = \frac{2}{\frac{16}{\theta^2}} = \frac{\theta^2}{8}$

$\Rightarrow E[X] = \frac{4}{\theta} \cdot \frac{\theta^2}{8} = \frac{\theta}{2}$

oppure si procede col calcolo diretto dell'integrale iniziale.

Pertanto $\bar{X}_m = \frac{\theta}{2} \Rightarrow \hat{\theta} = 2\bar{X}_m$

$$E[\hat{\Theta}] = E[2\bar{X}_m] = 2E[\bar{X}_m] = 2\mu = 2E[X] = 2\cdot\frac{\theta}{2} = \theta$$

$\hat{\Theta}$ è corretto

$$T_2 = \frac{3X_1 + \bar{X}_m}{8}$$

$$E[T_2] = \frac{1}{8}E[3X_1 + \bar{X}_m] = \frac{1}{8}\{3\underbrace{E[X_1]}_{E[X]} + \underbrace{E[\bar{X}_m]}_{\mu = E[X]}\} = \frac{1}{8}\cdot 4E[X]$$

$$= \frac{1}{2}E[X] = \frac{\theta}{4}$$

$\Rightarrow$ T_2 non è corretto.

$\Rightarrow$ $\hat{\Theta}$ è preferibile

E2)

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

se $x \leq 0$ $F(x) = 0$

se $0 < x \leq 1$ $F(x) = \int_{-\infty}^{0} \underbrace{f(t)}_{0}\,dt + \int_0^x \frac{1}{2}t^2\,dt = \frac{1}{2}\cdot\frac{1}{3}x^3 = \frac{1}{6}x^3$

se $1 < x \leq 2$ $F(x) = \int_{-\infty}^{0} \underbrace{f(t)}_{0}\,dt + \int_0^1 \frac{1}{2}t^2\,dt + \int_1^x \frac{5}{6}\,dt$

$$= \frac{1}{2}\cdot\frac{1}{3} + \frac{5}{6}(x-1) = \frac{5}{6}x - \frac{2}{3}$$

se $x > 2$ $F(x) = 1$

quindi

$$F(x) = \begin{cases} 0 & \text{se } x \leq 0 \\ \frac{1}{6}x^3 & \text{se } 0 < x \leq 1 \\ \frac{5}{6}x - \frac{2}{3} & \text{se } 1 < x \leq 2 \\ 1 & \text{se } x > 2 \end{cases}$$

$$var[X] = E[X^2] - E[X]^2$$

$$E[X^2] = \int_{-\infty}^{+\infty} x^2 f(x)\,dx = \int_0^1 x^2 \cdot \frac{1}{2} x^2\,dx + \int_1^2 \frac{5}{6} x^2\,dx$$

$$= \frac{1}{2} \cdot \frac{1}{5} + \frac{5}{6} \cdot \frac{1}{3}(8-1) = \frac{1}{10} + \frac{35}{18} = \frac{92}{45}$$

$$E[X] = \int_0^1 x \cdot \frac{1}{2} x^2\,dx + \int_1^2 \frac{5}{6} x\,dx$$

$$= \frac{1}{2} \cdot \frac{1}{4} + \frac{5}{6} \cdot \frac{1}{2}(4-1) = \frac{1}{8} + \frac{5}{4} = \frac{11}{8}$$

$$\Rightarrow var[X] = 443/2880$$

$$P\left[\frac{1}{2} \le X \le 4\right] = P\left[\frac{1}{2} \le X \le 2\right] = F(2) - F\left(\frac{1}{2}\right)$$

$$= 1 - \frac{1}{6}\left(\frac{1}{2}\right)^3 = 1 - \frac{1}{48} = \frac{47}{48}$$

se $Y \sim Unif.[-1, 1]$ $\quad E[Y] = \frac{1-1}{2} = 0 \quad \left(E[Y] = \frac{a+b}{2}\right)$

$$E[(X+Y)^2] = E[X^2 + 2XY + Y^2] = E[X^2] + 2E[XY] + E[Y^2]$$

$$\underset{\text{INDIP.}}{=} E[X^2] + 2E[X] \cdot \underbrace{E[Y]}_{=0} + E[Y^2]$$

$$= E[X^2] + E[Y^2]$$

$$var[Y] = \frac{1}{3} \qquad \left(var[Y] = \frac{(b-a)^2}{12}\right)$$

$$var[Y] = E[Y^2] - \underbrace{E[Y]^2}_{=0} = E[Y^2]$$

$$\Rightarrow E[(X+Y)^2] = \frac{92}{45} + \frac{1}{3} = \frac{107}{45}$$

Prova scritta del 10.12.2008 (fila 1)

C1) $X \sim N(\mu, \sigma^2)$ σ^2: $P[|X-\mu|>2]=0.1141$

$$P[|X-\mu|>2] = 1-P[|X-\mu|\le 2] = 1-P[-2\le X-\mu \le 2]$$

$$= 1-P\left[-\frac{2}{\sigma}\le Z\le \frac{2}{\sigma}\right] = 1-2P\left[0\le Z\le \frac{2}{\sigma}\right]$$

$$= 1-2\left\{P\left[Z\le\frac{2}{\sigma}\right]-\frac{1}{2}\right\} = 2-2P\left[Z\le\frac{2}{\sigma}\right]$$

$$\Rightarrow P\left[Z\le\frac{2}{\sigma}\right] = \frac{2-0.1141}{2} = 0.94295 \xrightarrow[TAB]{} z_\alpha = 1.58$$

$$\Rightarrow \frac{2}{\sigma} = 1.58 \Rightarrow \sigma = \frac{2}{1.58} = 1.2658 \Rightarrow \underline{\sigma^2 = 1{,}60230}$$

C2) $n = 100$ $p = 0.01$ guasto

$X \sim B(100, 0.01)$

$$P[X\ge 3] = 1-P[X<3] = 1-P[X\le 2] = 1-P[X=0]-P[X=1]-P[X=2]$$

$$P[X=0] = \underbrace{\binom{100}{0}}_{=1}(0.01)^0\cdot(0.99)^{100}$$

$$P[X=1] = \underbrace{\binom{100}{1}}_{=100}(0.01)^1\cdot(0.99)^{99}$$

$$P[X=2] = \underbrace{\binom{100}{2}}_{=5050}(0.01)^2\cdot(0.99)^{98}$$

sostituendo $\underline{P[X\ge 3] = 0.079373}$

C3)

$V \sim N(\mu=2, \sigma=1)$ $W = 5V^2$

$$\underline{E[W]} = 5E[V^2] = 5\left(var[V]+E[V]^2\right)$$

$$= 5(1+4) = \underline{25}$$

C4) La funzione di densità di X NON È NOTA.

$\mu_X = 80 \quad \sigma_X^2 = 25$

$m: \; P[|\bar{X}_m - 80| \leq 5] \geq 0.8$

Per la legge dei grandi numeri

$\varepsilon = 5$

$1-\delta = 0.8 \Rightarrow \delta = 0.2 \qquad m \geq \frac{\sigma^2}{\delta \varepsilon^2} = \frac{25}{25 \cdot 0,2} = 5$

QT)

$var\left[\frac{X}{\sigma_X} - \frac{Y}{\sigma_Y}\right] \geq 0 \Rightarrow \rho_{X,Y} \leq 1$

$\rho_{X,Y} = \frac{cov[X,Y]}{\sigma_X \cdot \sigma_Y}$ perciò $\rho_{X,Y} \leq 1 \sim cov[X,Y] \leq \sigma_X \cdot \sigma_Y$

$$0 \leq var\left[\frac{X}{\sigma_X} - \frac{Y}{\sigma_Y}\right] = var\left[\frac{X}{\sigma_X}\right] + var\left[\frac{Y}{\sigma_Y}\right] - 2 cov\left[\frac{X}{\sigma_X}, \frac{Y}{\sigma_Y}\right]$$

$$= \frac{1}{\sigma_X^2} \cdot \sigma_X^2 + \frac{1}{\sigma_Y^2} \cdot \sigma_Y^2 - 2 \cdot \frac{1}{\sigma_X \cdot \sigma_Y} cov[X,Y]$$

$$= 2 - \frac{2}{\sigma_X \sigma_Y} cov[X,Y]$$

allora

$-\frac{2}{\sigma_X \sigma_Y} cov[X,Y] + 2 \geq 0$ cioè $cov[X,Y] \leq \sigma_X \cdot \sigma_Y$

e quindi $\rho_{X,Y} \leq 1$.

E1)

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$1 = \int_{\frac{\theta}{2}}^{+\infty} \frac{k}{x^4}\,dx = k \cdot \left(-\frac{1}{3}\right)\left[x^{-3}\right]_{\frac{\theta}{2}}^{+\infty} = -\frac{k}{3} \cdot \left(-\frac{1}{\frac{\theta^3}{8}}\right)$$

$$= \frac{8}{3}\frac{k}{\theta^3} \Rightarrow k = \frac{3}{8}\theta^3$$

$M_1' = \mu_1'$ dove $M_1' = \bar{X}_n$ e $\mu_1' = \mu = E[X]$

$$E[X] = \int_{\frac{\theta}{2}}^{+\infty} x \cdot \frac{3}{8}\theta^3 \cdot \frac{1}{x^4}\, dx = \frac{3}{8}\theta^3 \cdot \left(-\frac{1}{2}\cdot\frac{1}{x^2}\right)_{\frac{\theta}{2}}^{+\infty}$$

$$= \frac{3}{16}\theta^3 \cdot \frac{4}{\theta^2} = \frac{3}{4}\theta$$

$\Rightarrow \bar{X}_n = \frac{3}{4}\theta$ cioè $\underline{\hat{\theta} = \frac{4}{3}\bar{X}_n}$

$$\underline{E[\hat{\theta}]} = E\left[\frac{4}{3}\bar{X}_n\right] = \frac{4}{3}E[\bar{X}_n] = \frac{4}{3}\mu = \frac{4}{3}E[X] = \frac{4}{3}\cdot\frac{3}{4}\theta = \underline{\theta}$$

$\underline{\hat{\theta} \text{ è corretto}}$

$$MSE[\hat{\theta}] = var[\hat{\theta}] + \overset{=0}{D^2[\hat{\theta}]} = var\left[\frac{4}{3}\bar{X}_n\right] = \frac{16}{9} var[\bar{X}_n]$$

$$= \frac{16}{9}\frac{\sigma^2}{n}$$

$$\sigma^2 = E[X^2] - E[X]^2$$

$$E[X^2] = \int_{\frac{\theta}{2}}^{+\infty} x^2 \cdot \frac{3}{8}\theta^3 \cdot \frac{1}{x^4}\, dx = \frac{3}{8}\theta^3 \cdot \left(-\frac{1}{x}\right)_{\frac{\theta}{2}}^{+\infty} = \frac{3}{8}\theta^3 \cdot \frac{2}{\theta} = \frac{3}{4}\theta^2$$

$$\sigma^2 = \frac{3}{4}\theta^2 - \frac{9}{16}\theta^2 = \frac{3}{16}\theta^2$$

$$\Rightarrow \underline{MSE[\hat{\theta}]} = \frac{16}{9}\cdot\frac{3}{16}\frac{\theta^2}{n} = \underline{\frac{\theta^2}{3n}}$$

E2)

se $X = -1$ $\quad Y = 2$ $\qquad f_X(-1) = \frac{1}{2}$

se $X = 0$ $\quad Y = 1$ $\qquad f_X(0) = \frac{1}{3}$

se $X = 1$ $\quad Y = 2$ $\qquad f_X(1) = \frac{1}{6}$

sono le marginali della X.

costruisco la tabella a doppia entrata

	$X=-1$	$X=0$	$X=1$	f_Y
$Y=1$	0	•	0	
$Y=2$	•	0	•	
f_X	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$	1

Si ricava subito $f_{X,Y}(-1,1) = f_{X,Y}(0,2) = f_{X,Y}(1,1) \equiv 0$

e che $f_{X,Y}(-1,2) = \frac{1}{2}$

$f_{X,Y}(0,1) = \frac{1}{3}$

$f_{X,Y}(1,2) = \frac{1}{6}$

pertanto $f_Y(1) = \frac{1}{3}$ e $f_Y(2) = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$ (o subito da $1 - \frac{1}{3}$)

X e Y sono dipendenti

$$\underset{\underset{0}{\equiv}}{f_{X,Y}(-1,1)} \neq \underset{\underset{\frac{1}{2}}{\parallel}}{f_X(-1)} \cdot \underset{\underset{\frac{1}{3}}{\parallel}}{f_Y(1)}$$

$$\rho_{X,Y} = \frac{E[XY] - E[X]\cdot E[Y]}{\sigma_X \cdot \sigma_Y}$$

$E[X] = -1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{6} = -\frac{1}{3}$

$E[Y] = 1 \cdot \frac{1}{3} + 2 \cdot \frac{2}{3} = \frac{5}{3}$

$E[XY] = (-1) \cdot 2 \cdot \frac{1}{2} + 1 \cdot 2 \cdot \frac{1}{6} = -\frac{2}{3}$

$E[X^2] = 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{6} = \frac{2}{3} \Rightarrow \sigma_X^2 = \frac{2}{3} - \frac{1}{9} = \frac{5}{9} \Rightarrow \sigma_X = \frac{\sqrt{5}}{3}$

$E[Y^2] = 1 \cdot \frac{1}{3} + 4 \cdot \frac{2}{3} = 3 \Rightarrow \sigma_Y^2 = 3 - \frac{25}{9} = \frac{2}{9} \Rightarrow \sigma_Y = \frac{\sqrt{2}}{3}$

$\Rightarrow \rho_{X,Y} = \left(-\frac{2}{3} + \frac{5}{9}\right) / (\sqrt{10}/9) = -\frac{1}{\sqrt{10}}$