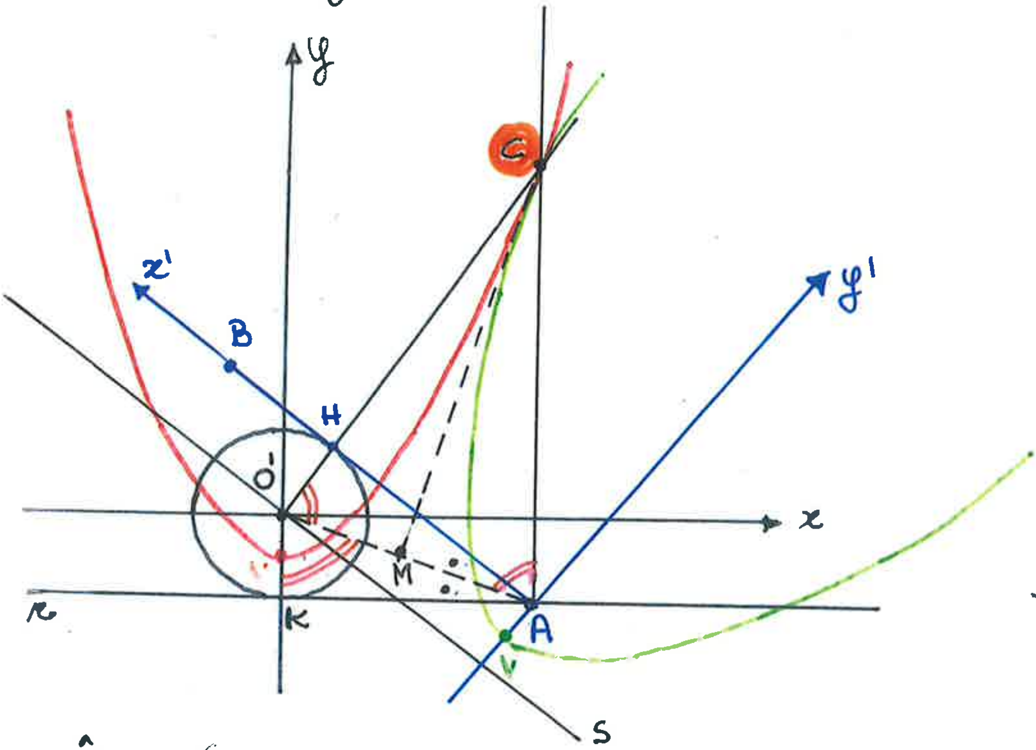
Esercizio: Determinare base e rulletta di un'asta AB tangente ad una circonferenza fissa e con l'estremo A vincolato ad una guida orizzontale (pag. 48 LIBRO)



$H\hat{A}K = 2\alpha$

$K\hat{O'}A = A\hat{O'}H$ (r e AB tangenti alla circonferenza)

$K\hat{O'}A = O'\hat{A}C$ (alterni interni)

$\Rightarrow$ O'CA è un triangolo isoscele $\Rightarrow$ $\overline{CO'} = \overline{CA}$.

Nel rif. O'xy C si mantiene a distanza costante dalla retta r (direttrice) e dal punto fisso O' (fuoco).

$\Rightarrow$ la base è una parabola.

Nel rif. A x'y' C si mantiene a distanza costante dalla retta s passante per O' e parallela ad AB (direttrice) e dal punto A (fuoco).

$\Rightarrow$ La rulletta è una parabola.

N.B.: I vertici delle parabole si trovano a metà della distanza tra fuoco e direttrice.

Determiniamo le equazioni della base e della rulletta

In $O'xy$ detto $2\alpha = H\hat{A}K$, $\overline{O'H} = \overline{O'K} = R$

$$\overline{O'A} = \frac{R}{\operatorname{sen}\alpha} \Rightarrow \overline{KA} = R \operatorname{cotg}\alpha$$

- $x_C = x_A = R \operatorname{cotg}\alpha$

I triangoli $O'\hat{K}A$ e $C\hat{M}A$ sono simili: $\overline{O'A} : \overline{CA} = \overline{O'K} : \overline{MA}$, $\overline{MA} = \frac{1}{2}\overline{O'A}$

$$\overline{CA} = \frac{R}{2\operatorname{sen}^2\alpha} = \frac{R}{2}(1+\operatorname{cotg}^2\alpha)$$

- $y_C = \frac{R}{2}(1+\operatorname{cotg}^2\alpha) - R$

$$\operatorname{cotg}\alpha = \frac{x_C}{R} \Rightarrow y_C = \frac{R}{2}\left(1+\frac{x_C^2}{R^2}\right) - R \Rightarrow \boxed{y = \frac{1}{2R}x^2 - \frac{R}{2}}$$ BASE

In $Ax'y'$

- $x'_C = x'_H = \overline{AH} = \overline{KA} = R \operatorname{cotg}\alpha = x_C$

Il triangolo $C\hat{H}A$ è rettangolo in H.

$$\overline{CH} = \sqrt{\overline{CA}^2 - \overline{AH}^2} = \sqrt{\left(\frac{R}{2\operatorname{sen}^2\alpha}\right)^2 - (R\operatorname{cotg}\alpha)^2} = \frac{R\cos 2\alpha}{2\operatorname{sen}^2\alpha} =$$

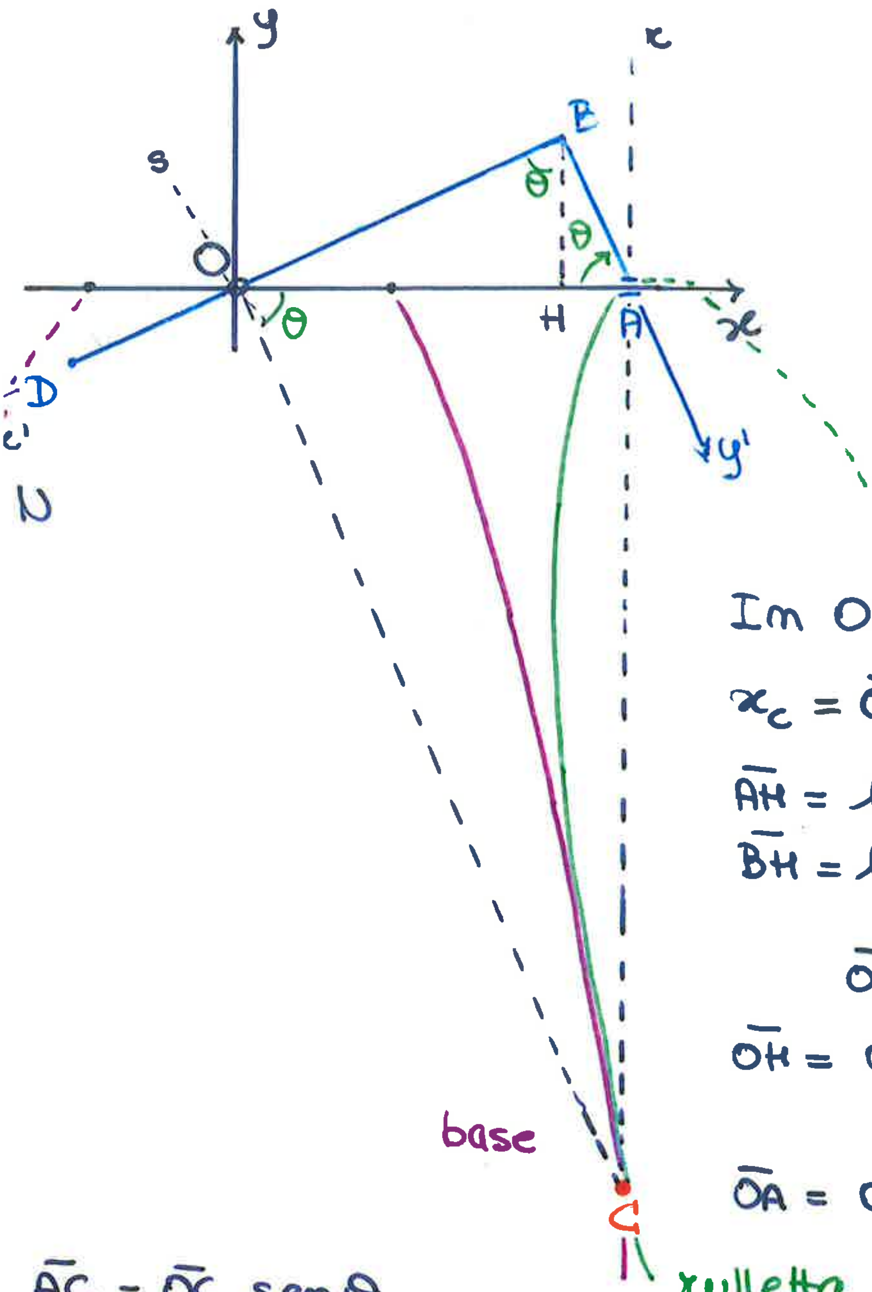
$$= \frac{R}{2}(\operatorname{cotg}^2\alpha - 1)$$

- $y'_C = \overline{CH} = \frac{R}{2}(\operatorname{cotg}^2\alpha - 1)$

$$\operatorname{cotg}\alpha = \frac{x'_C}{R} \Rightarrow y'_C = \frac{R}{2}\left(\frac{x'^2_C}{R^2} - 1\right) \Rightarrow \boxed{y' = \frac{1}{2R}x'^2 - \frac{R}{2}}$$ RULLETTA

N.B.: situazione completamente diversa se il disco rotola senza strisciare m. x..

Esercizio Determinare base e rulletta di un'asta fatta ad L: $A\hat{B}D$ avente il vertice A scorrevole su Ox e vincolata a passare per O, origine di Oxy.



DATI:

$\overline{AB} = \ell$

$DB = L$

L'asta ha 1 g. di libertà

$q = \theta \qquad O\hat{A}B = \theta$

$0 \leq \theta \leq \theta_L = \text{arc tg}\left(\frac{L}{\ell}\right)$

Per Chasles $s \cap r = C$

In Oxy: base

$x_c = \overline{OA} \quad , \quad y_c = -\overline{AC}$

$\overline{AH} = \ell \cos\theta$

$\underbrace{\overline{BH} = \ell \operatorname{sen}\theta \quad , \quad \overline{BH} = \overline{OB}\cos\theta}$

$\overline{OB} = \ell \operatorname{tg}\theta$

$\overline{OH} = \overline{OB}\operatorname{sen}\theta = \ell \dfrac{\operatorname{sen}^2\theta}{\cos\theta}$

$\overline{OA} = \overline{OH} + \overline{HA} = \dfrac{\ell}{\cos\theta}$

$\overline{AC} = \overline{OC}\operatorname{sen}\theta$

$\overline{OA} = \overline{OC}\cos\theta \quad \Rightarrow \quad \overline{AC} = \overline{OA}\operatorname{tg}\theta = \dfrac{\ell \operatorname{sen}\theta}{\cos^2\theta}$

$$\begin{cases} x_c = \dfrac{\ell}{\cos\theta} \\ y_c = -\ell\dfrac{\operatorname{sen}\theta}{\cos^2\theta} \end{cases}$$

Per determinare l'eq. della base bisogna eliminare θ

$x^2 = \dfrac{\ell^2}{\cos^2\theta} \quad \Rightarrow \quad \operatorname{sen}^2\theta = 1 - \dfrac{\ell^2}{x^2}$

$$y_c = -\ell\,(\pm)\sqrt{1-\frac{\ell^2}{x_c^2}}\cdot\frac{x_c^2}{\ell^2} = \mp\frac{x_c^2}{\ell}\sqrt{\frac{x_c^2-\ell^2}{x_c^2}} = -\frac{|x_c|}{\ell}\sqrt{x_c^2-\ell^2}$$

$$\Rightarrow \quad \underline{y = -\frac{|x|}{\ell}\sqrt{x^2-\ell^2}} \quad \text{base} \qquad x<-\ell\,,\ x>\ell$$

In $Bx'y'$ rulletta

$$x'_c = \overline{OB} \quad ; \quad y'_c = \overline{OC} \quad \Rightarrow \begin{cases} x'_c = \ell\,\mathrm{tg}\,\theta \\ y'_c = \dfrac{\ell}{\cos^2\theta} \end{cases}$$

$$x'^2_c = \ell^2\frac{\sin^2\theta}{\cos^2\theta} = \ell^2\frac{(1-\cos^2\theta)}{\cos^2\theta} = \frac{\ell^2}{\cos^2\theta} - \ell^2 = \ell y'_c - \ell^2$$

$$y'_c = \frac{1}{\ell}x'^2_c + \ell$$

$$\Rightarrow \quad \underline{y' = \frac{1}{\ell}x'^2 + \ell} \quad \text{rulletta (parabola).}$$

Abbiamo visto che θ è limitato.

Se supponiamo che $\overline{DB} = L = \sqrt{3}\,\ell \Rightarrow \theta_L = \frac{\pi}{3}$.

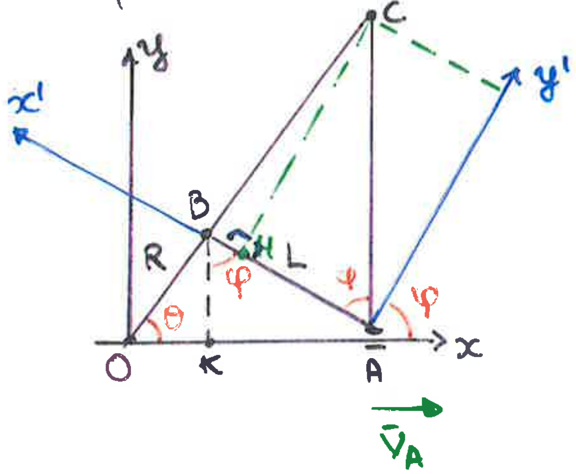
<u>Esercizio</u>. Determinare il C.I.R. dell'asta AB avente A scorrevole su Ox e B incernierato nell'estremo B dell'asta OB incernierata a sua volta in O.

C = retta $Ay \cap$ retta OB

$\bar{v}_B \perp \overline{OB}$

N.B. $\overline{OB} \not\perp \overline{AB}$

In questo caso det. base e rulletta. (eq. parametriche



base Oxy

$$\begin{cases} x_c = \overline{OA} \\ y_c = \overline{CA} \end{cases}$$

rulletta $A x' y'$

$$\begin{cases} x'_c = \overline{AH} \\ y'_c = \overline{CH} \end{cases}$$

$$\begin{cases} \overline{BK} = R \operatorname{sen} \theta \\ \overline{BK} = L \cos \varphi \end{cases} \Rightarrow \operatorname{sen} \theta = \frac{L}{R} \cos \varphi \ : \text{legame}$$

$$\overline{OK} = R \cos \theta = R \sqrt{1 - \operatorname{sen}^2 \theta} = \sqrt{R^2 - L^2 \cos^2 \varphi}$$

$$\overline{OA} = \overline{OK} + \overline{KA} = \sqrt{R^2 - L^2 \cos^2 \varphi} + L \operatorname{sen} \varphi$$

similitudine

$$\overline{OK} : \overline{OA} = \overline{BK} : \overline{CA} \Rightarrow \overline{CA} = \frac{\overline{OA} \cdot \overline{BK}}{\overline{OK}}$$

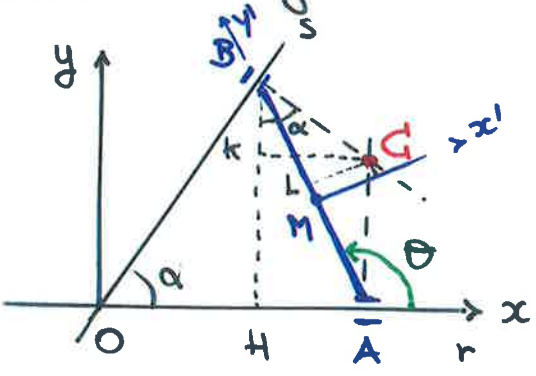
$$\begin{cases} x_c = L \operatorname{sen} \varphi + \sqrt{R^2 - L^2 \cos^2 \varphi} \\ y_c = \dfrac{\left(L \operatorname{sen} \varphi + \sqrt{R^2 - L^2 \cos^2 \varphi}\right) L \cos \varphi}{\sqrt{R^2 - L^2 \cos^2 \varphi}} \end{cases}$$

$$\overline{AH} = \overline{AC} \cos \varphi$$

$$\overline{CH} = \overline{AC} \operatorname{sen} \varphi$$

$$\begin{cases} x'_c = y_c \cos \varphi \\ y'_c = y_c \operatorname{sen} \varphi \end{cases}$$

Asta $\overline{AB} = \ell$ avente gli estremi A e B scorrevoli su due guide formanti un angolo $\alpha \neq \pi$. Determinare base e rulletta.



$\alpha \leq \theta \leq \pi$

$\overline{BH} = \overline{AB}\ \sin(\pi - \theta)$

$= \ell \sin\theta$

$\overline{BH} = \overline{OB} \sin\alpha$

$\Rightarrow \overline{OB} = \dfrac{\ell \sin\theta}{\sin\alpha}$

$\overline{OH} = \overline{OB} \cos\alpha = \ell \sin\theta \operatorname{cotg}\alpha$

$\overline{HA} = \overline{AB} \cos(\pi - \theta) = -\ell \cos\theta$

- $x_C = \ell \sin\theta \operatorname{cotg}\alpha - \ell \cos\theta$

$\overline{BK} = \overline{BC} \cos\alpha$

$\overline{KC} = \overline{BC} \sin\alpha$

$\Rightarrow \overline{BK} = \overline{KC} \operatorname{cotg}\alpha = \overline{AH} \operatorname{cotg}\alpha = -\ell \cos\theta \operatorname{cotg}\alpha$

- $y_C = \ell \sin\theta + \ell \cos\theta \operatorname{cotg}\alpha$

$x_C^2 + y_C^2 = \ell^2 + \ell^2 \operatorname{cotg}^2\alpha = \dfrac{\ell^2}{\sin^2\alpha}$

BASE

circonferenza di centro O e raggio $r = \dfrac{\ell}{\sin\alpha}$

se $\theta = \alpha$ $\begin{cases} x_C = \ell \sin\alpha \dfrac{\cos\alpha}{\sin\alpha} - \ell \cos\alpha = 0 \\ y_C = \ell \sin\alpha + \ell \dfrac{\cos^2\alpha}{\sin\alpha} = \dfrac{\ell}{\sin\alpha} \end{cases}$

se $\theta = \pi$ $\begin{cases} x_C = \ell \\ y_C = -\ell \operatorname{cotg}\alpha \end{cases}$

Introdotto un sistema di riferimento solidale con $\overline{AB}$ cioè $Mx'y'$: My' supporto di $\overline{AB}$ e $Mx' \perp$ ad $\overline{AB}$.

$x'_C = \overline{CA} \sin\left(\theta - \frac{\pi}{2}\right) = -y_C \cos\theta$

$y'_C = \overline{LA} - \overline{MA}$

$$\overline{LA} = \overline{CA}\cos\left(\theta - \frac{\pi}{2}\right) = y_C \sin\theta$$

$$\overline{MA} = \frac{\ell}{2}$$

$$y_C = y_C \sin\theta - \frac{\ell}{2}$$

- $x'_C = -\ell\sin\theta\cos\theta - \ell\cos^2\theta\,\mathrm{cotg}\,\alpha$
- $y'_C = \ell\sin^2\theta + \ell\sin\theta\cos\theta\,\mathrm{cotg}\,\alpha - \frac{\ell}{2}$

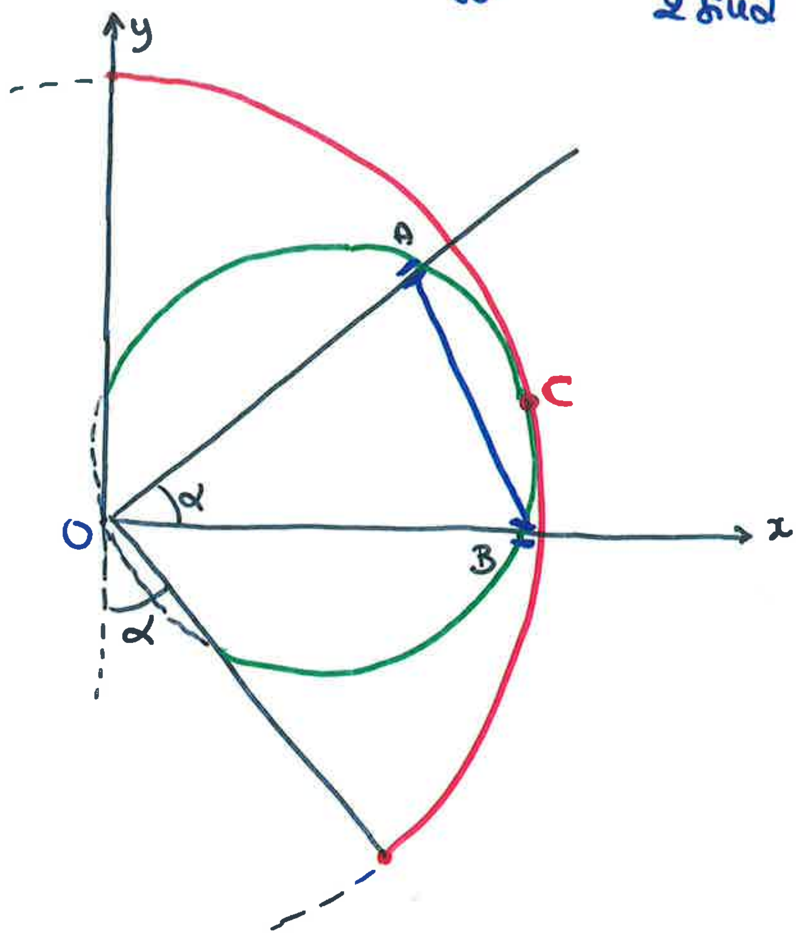
$$x'^2_C + y'^2_C - \ell\,\mathrm{cotg}\,\alpha\, x'_C - \frac{\ell^2}{4} = 0$$

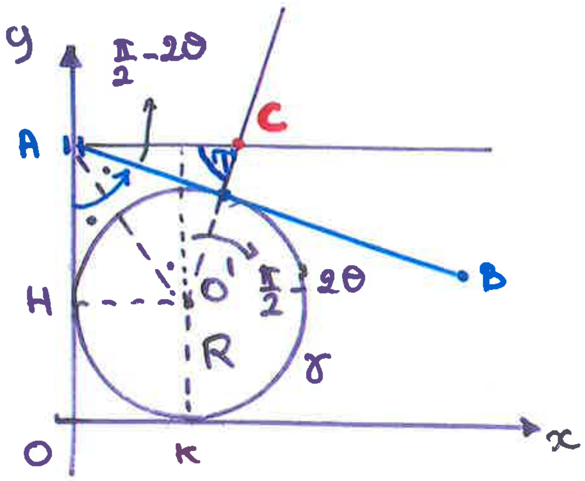
RULLETTA

CIRCONFERENZA di centro Q

$Q\left(-\frac{\ell}{2}\mathrm{cotg}\,\alpha\, ,0\right)$ e

raggio $r' = \frac{\ell}{2\sin\alpha}$





γ fissa

$\overline{AB}$: A scorrevole su Oy appoggiata a γ

$O\hat{A}B = 2\theta$

$H\hat{A}O' = O'\hat{A}B = \theta$

$$x_c = R + \overline{O'C}\cos 2\theta$$

$$y_c = R + \overline{AH}$$

$$\overline{AH} = \overline{O'C}\sin 2\theta$$

$$\overline{AH} = \overline{AT} = \overline{AO'}\cos\theta$$

ma

$$\overline{O'H} = \overline{AO'}\sin\theta \Rightarrow \overline{AO'} = \frac{R}{\sin\theta}$$

$$\Rightarrow \overline{AH} = R\,\mathrm{cotg}\,\theta$$

$$\Rightarrow \overline{O'C} = \frac{\overline{AH}}{\sin 2\theta} = \frac{R\,\mathrm{cotg}\,\theta}{\sin 2\theta}$$

$$\begin{cases} x_c = R + R\,\mathrm{cotg}\,\theta\,\mathrm{cotg}\,2\theta \\ y_c = R + R\,\mathrm{cotg}\,\theta \end{cases}$$

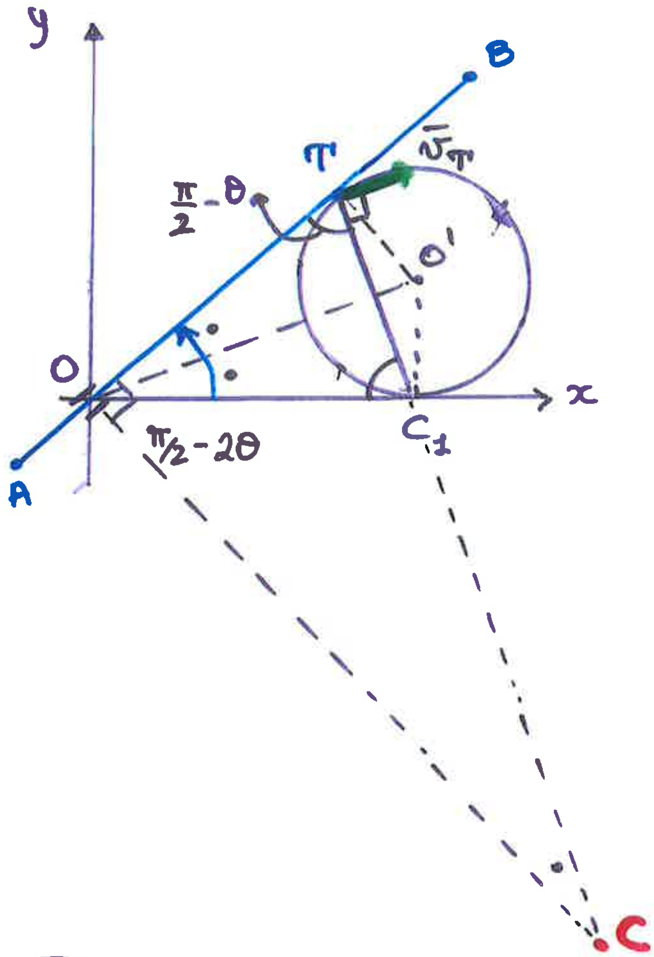
$$\mathrm{cotg}\,\theta\,\mathrm{cotg}\,2\theta = \frac{1}{\mathrm{tg}\,\theta}\cdot\frac{1-\mathrm{tg}^2\theta}{2\,\mathrm{tg}\,\theta} = \frac{1}{2}\left(\mathrm{cotg}^2\theta - 1\right)$$

$$\begin{cases} x_c = R + \frac{R}{2}\mathrm{cotg}^2\theta - \frac{R}{2} = \frac{R}{2}\left(1+\mathrm{cotg}^2\theta\right) \\ y_c = R + R\,\mathrm{cotg}\,\theta \end{cases}$$

$$\mathrm{cotg}\,\theta = \frac{y-R}{R}$$

$$x = \frac{R}{2}\left[1 + \frac{(y-R)^2}{R^2}\right] = \frac{1}{2R}\left(y^2 - 2Ry + 2R^2\right)$$

$$\Rightarrow \boxed{x = \frac{1}{2R}y^2 - y + R}$$ PARABOLA



asta $\overline{AB}$:

- passante per O

$B\hat{O}x = 2\theta$

$T\hat{O}O' = O'\hat{O}x = \theta$

- appoggiata a $\mathcal{D}$ che r.s.s. su Ox

$T' \in \overline{AB}$

$T'' \in \mathcal{D}$ $\quad T' = T'' = T$

$\bar{v}_{T'} = \bar{v}_{T''}$ per r.s.s.

$$\bar{v}_{T''} = \underbrace{\bar{v}_{C_1}}_{\equiv \bar{0}} + \bar{\omega} \times (T'' - C_1)$$

$\Rightarrow \bar{v}_{T''} \perp (T'' - C_1)$

$\overline{OT} = \overline{OC_1} = \overline{OO'} \cos\theta$

$\overline{O'C_1} = \overline{OO'} \sin\theta \quad \Rightarrow \overline{OO'} = \dfrac{R}{\sin\theta}$

$\Rightarrow \overline{OT} = R \operatorname{cotg}\theta$

$$\begin{cases} x_T = \overline{OT} \cos 2\theta = R \operatorname{cotg}\theta \cos 2\theta \\ y_T = \overline{OT} \sin 2\theta = R \operatorname{cotg}\theta \sin 2\theta \end{cases}$$

$\overline{OT} = \overline{TC} \sin\theta \quad \Rightarrow \overline{TC} = \dfrac{\overline{OT}}{\sin\theta} = \dfrac{R\cos\theta}{\sin^2\theta}$

$\overline{OC} = \overline{TC} \cos\theta = R \operatorname{cotg}^2\theta$

$$\begin{cases} x_c = \overline{OC} \cos\left(\frac{\pi}{2} - 2\theta\right) = R \operatorname{cotg}^2\theta \sin 2\theta \\ y_c = -\overline{OC} \sin\left(\frac{\pi}{2} - 2\theta\right) = -R \operatorname{cotg}^2\theta \cos 2\theta \end{cases}$$

se $\theta = \dfrac{\pi}{6}$ $\quad \overline{OC} = 3R$

$$\begin{cases} x_c = 3R \frac{\sqrt{3}}{2} \\ y_c = 3R \frac{1}{2} \end{cases} \qquad \boxed{x^2 + y^2 = 9R^2}$$

CIRCONFERENZA