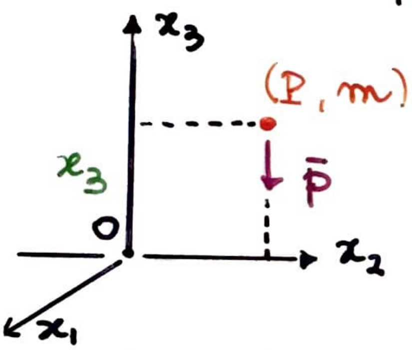
Esempi di forze conservative

1) **FORZA PESO** $\quad \vec{p} = m\vec{g}$ vettore costante (in prossimità del suolo)

$$L = \oint \vec{p} \cdot d\vec{x} = 0$$

$\Rightarrow \exists\, \mathcal{U} = \mathcal{U}(P)$ potenziale : $\quad \bar{p} = \operatorname{grad} \mathcal{U}$.

Scelto un rif. cart. ortogonale $O x_1 x_2 x_3$

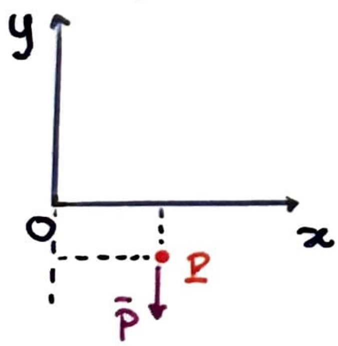


$$\bar{p} = -mg\, \bar{\imath}_3$$

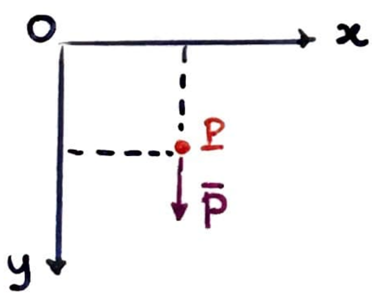
$$\frac{\partial \mathcal{U}}{\partial x_3} = -mg \qquad \frac{\partial \mathcal{U}}{\partial x_1} = 0 \qquad \frac{\partial \mathcal{U}}{\partial x_2} = 0$$

$\Rightarrow \underline{\underline{\mathcal{U} = \mathcal{U}(x_3) = -mg\, x_3 + \text{costante}}}$

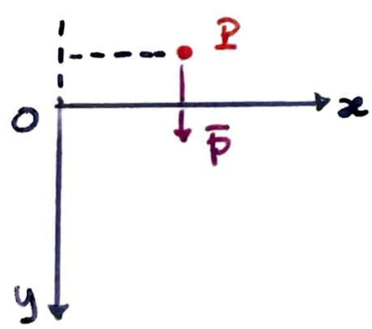
x_3: quota di P rispetto ad O.



$$\mathcal{U} = -mg(-y) + c = mgy + c$$

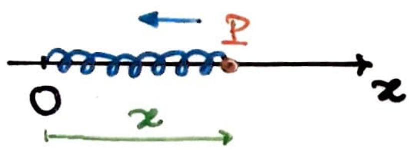


$$\mathcal{U} = mg\, y + c$$



$$\mathcal{U} = mg(-y) + c = -mg\, y + c$$

2) FORZA ELASTICA : $\bar{F} = -k(P-O)$, $k>0$



$\bar{F} = -k\vec{x}$ molla ideale

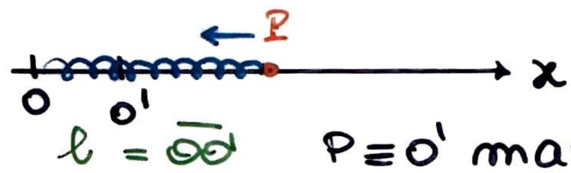
$\Rightarrow$ lunghezza a riposo nulla

$$L = \oint -k\vec{x}\cdot d\vec{x} = -\frac{k}{2}\oint d\vec{x}^2 = 0$$

$$U = U(x) = -\frac{1}{2}k\,x^2 + \text{costante}$$

se la molla non ha lunghezza a riposo nulla :

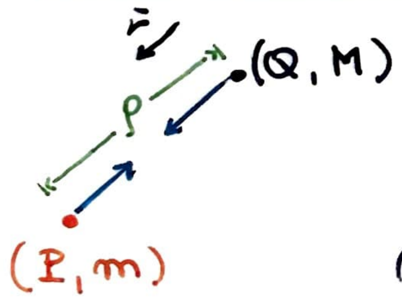
$$\bar{F} = -k(x-\ell)\,\bar{\imath}$$



$P \equiv O'$ massima compressione

$$U = U(x) = -\frac{1}{2}(x-\ell)^2 + c$$

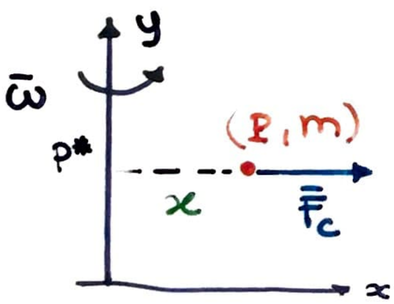
3) FORZA DI ATTRAZIONE NEWTONIANA



$$\bar{F}(P) = -\frac{k\,m\,M}{\rho^2}\,\text{vers}\underbrace{(P-Q)}_{``\vec{r}"}$$

$$U = U(\rho) = \int -\frac{k\,m\,M}{\rho^2}\,d\rho = \frac{k\,m\,M}{\rho} + c$$

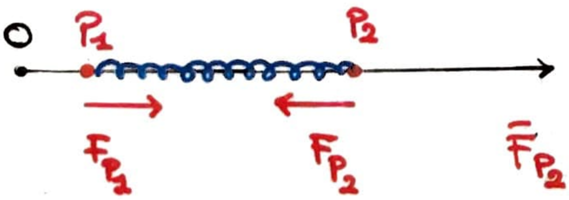
4) FORZA CENTRIFUGA (esiste in rif. non inerziali)



$\bar{F}(P) = m\omega^2(P-P^*)$ $\quad\bar{\omega}$ = costante

$$U = U(x) = \frac{1}{2}m\omega^2 x^2 + c$$

$(P_1, m_1) + (P_2, m_2)$



$\bar{F}_{P_2} = -\bar{F}_{P_1}$

$\bar{F}_{P_2} = -k(P_2 - P_1) \quad , k>0$

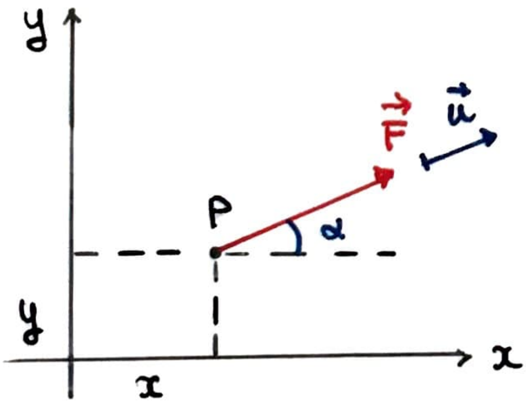
$\bar{F}_{P_1} = -k(P_1 - P_2), \quad k>0$

$$dU = dL = \bar{F}_{P_2} \cdot dP_2 + \bar{F}_{P_1} \cdot dP_1 = -k(P_2 - P_1) \cdot d(P_2 - O) - k(P_1 - P_2) \cdot d(P_1 - O)$$

$$= -k(P_2 - P_1) \cdot [d(P_2 - O) - d(P_1 - O)]$$

$$= -k(P_2 - P_1) \cdot d(P_2 - P_1) = -\frac{k}{2} d|P_2 - P_1|^2$$

$$\Rightarrow \mathcal{U} = -\frac{1}{2} k |P_2 - P_1|^2 + c$$



(P, m)

$\alpha =$ costante

$\vec{F} = F\bar{u}$

$\vec{u} = \cos\alpha\, \bar{i} + \sin\alpha\, \bar{j}$

$P(x, y)$

$dP = dx\, \bar{i} + dy\, \bar{j}$

$$dU = dL = F\cos\alpha\, dx + F\sin\alpha\, dy \quad e \quad \oint dL = 0$$

$$\frac{\partial U}{\partial x} = F\cos\alpha \quad , \quad \frac{\partial U}{\partial y} = F\sin\alpha$$

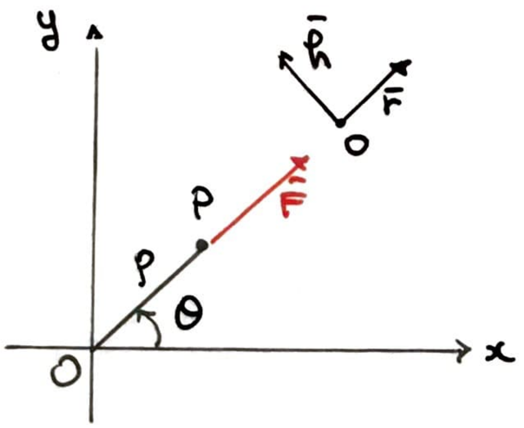
$$\Rightarrow \mathcal{U}(x, y) = F\cos\alpha\, x + F\sin\alpha\, y + c$$

(P, m)

$\vec{F} = F\vec{r}$

$(P-O) = \rho\vec{r}$

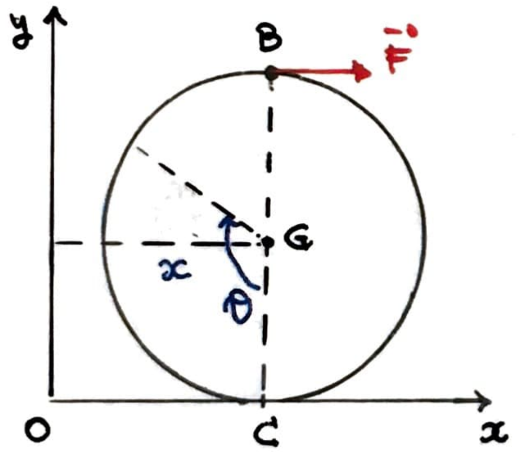
$dP = d\rho\,\vec{r} + \rho\, d\theta\, \vec{h}$



$dU = dL = \bar{F} \cdot dP = F\,d\rho$

$\oint dL = 0$

$\Rightarrow U(\rho) = \int F\,d\rho = F\rho + c$



$D: m, R$

$\dot{x} = R\dot{\theta}$ r.s.s.

$\bar{F} = F\vec{i}$

$dB = ?$

$\bar{v}_B = \underset{\overset{\parallel}{0}}{\bar{v}_C} + \bar{\omega} \times (B-C)$

$= -\dot{\theta}\bar{k} \times 2R\bar{j} = 2R\dot{\theta}\vec{i}$

$dB = 2R\,d\theta\,\bar{i}$

$dU = dL = \bar{F} \cdot dB = F\bar{i} \cdot 2R\,d\theta\,\bar{i} = 2RF\,d\theta$

$\oint dL = 0$

$\Rightarrow U(\theta) = 2RF\theta + c$

se poi uso x al posto di θ: $(x = R\theta + c)$

$\Rightarrow U(x) = 2Fx + c$

POTENZIALE DI UNA COPPIA AGENTE SU C. R.

$$dL = \sum_{s=1}^{N} \bar{F}_s \cdot dP_s$$

Per C.R. $\bar{v}_P = \bar{v}_{O'} + \bar{\omega} \times (P - O')$

$$dP = dO' + \bar{\omega}\, dt \times (P - O')$$

$$dL = \sum_{s=1}^{N} \bar{F}_s \cdot dP_s = \sum_{s=1}^{N} \bar{F}_s \cdot dO' + \sum_{s=1}^{N} \bar{F}_s \cdot [\bar{\omega}\, dt \times (P_s - O')]$$

$$= \left(\sum_{s=1}^{N} \bar{F}_s\right) \cdot dO' + \sum_{s=1}^{N} \bar{F}_s \cdot [(O' - P_s) \times \bar{\omega}\, dt]$$

$$= \vec{R} \cdot dO' + \left(\sum_{s=1}^{N} \bar{F}_s \times (O' - P_s)\right) \cdot \bar{\omega}\, dt$$

$$= \vec{R} \cdot dO' + \vec{M}_{O'} \cdot \bar{\omega}\, dt$$

Se su C.R. agisce una coppia $\Rightarrow \bar{R} = \bar{0}$

$dL = \bar{M}_{O'} \cdot \bar{\omega}\, dt$ ma il mom. non dipende dal polo

$$= \bar{M} \cdot \bar{\omega}\, dt$$

- se $\bar{M} = M \bar{k}$ e $\bar{\omega} = \dot{\theta} \vec{k}$

$dL = M \bar{k} \cdot \dot{\theta} \bar{k}\, dt = M\, d\theta$ $\qquad \oint dL = 0$

$dL = dU$

$U = \int M\, d\theta = M\theta + c \quad \Rightarrow \mathcal{U}(\theta) = M\theta + c$

- se $\vec{M} = M(\theta) \bar{k}$ e $\bar{\omega} = \dot{\theta} \bar{k}$

$dL = M(\theta) \bar{k} \cdot \dot{\theta} \vec{k}\, dt = M(\theta)\, d\theta$

$dL = dU$

$$\boxed{\mathcal{U}(\theta) = \int M(\theta)\, d\theta + c}$$