

MR 2010

TEMI DI ESAME - SOLUZIONI

16.06.2010 - FILA 1

Es. 1

$$1) l = \frac{R}{3} s$$

$$2) s = \frac{3}{10} m/R^2$$

$$l = \frac{1}{10} m/R$$

$$3) I_{\text{eff}} = \frac{22}{15} mR^2$$

Es. 2

$$1) \theta_0 = \pi/6$$

$$2) \vec{\Phi}_A = mg \frac{\sqrt{3}}{2} \vec{i}$$

$$\vec{\Phi}_N = \frac{mg}{2} \vec{j}$$

$$\vec{\Phi}_B = \frac{5}{4} mg \vec{j}$$

$$3) \vec{\Phi}_H = (0, \frac{mg}{2})$$

Es. 3

$$1) T = \frac{1}{2} m \left( \frac{1}{3} \dot{\theta}^2 + \dot{\varphi}^2 \right)$$

$$2) L = T + U \text{ dove}$$

$$U = mgR \left( \frac{3}{2} \cos \varphi + 2 \sin \theta \cos \varphi - \sin^2 \theta \right) + c$$

$$3) T + V = E, \quad V = -U, \quad E = T_0 + V_0.$$

≠ perché vincoli non e' holo, e forte conservative.

Ex. 1

1)  $l = \frac{R}{21} s$

2)  $s = \frac{4}{16} m/R^2$

$l = \frac{1}{48} m/R$

3)  $I_{\Delta P} = \frac{31}{36} mR^2$

Ex. 2

1)  $(2L, 0); (2L, \pi)$

2) nou ci sunt pozitii de cufine de echilibru

3)  $(2L, 0)$  stabile

$(2L, \pi)$  instabil.

Ex. 3

1)  $\pi = \frac{3}{4} \omega R^2 \dot{\theta}^2 + \frac{8}{3} \omega R^2 \dot{\varphi}^2$

2)  $U = 2mgR \cos \varphi + 2mgR \theta - \frac{1}{2} \frac{mg}{R} (R^2 \theta^2 - 8R^2 \theta \sin \varphi + 8Rc \sin \varphi + 2R \theta c) + \text{constanta}$

daca ie pt 0' la coordonate  $(R\theta + c, R)$

3)  $\frac{3}{2} R^2 \ddot{\theta} - 2mgR + g(R\theta - 4R \sin \varphi + c) = 0$

$\frac{16}{3} R^2 \ddot{\varphi} + 2gR \sin \varphi + g(4R\theta \cos \varphi - 4R \sin \varphi - 4c \cos \varphi) = 0$

4)  $\vec{F}_c(t) = mR\ddot{\theta} + mg(\theta - 4 \sin \varphi - 1 + \frac{c}{R}) \vec{i} +$

$mg(2 + 4 \cos \varphi) \vec{j}$

07.09.2010 - FILA 1

Es. 1

$$1) \cdot y_G = \frac{(15\pi^2 + 28)R}{21\pi}$$

$$2) I_C = \frac{(16\pi^2 + 45)mR^2}{84}$$

Es. 2

$$1) \alpha = 2 \rightarrow s = 0 \Rightarrow (0, \frac{\pi}{2})$$

$$2) \vec{\Phi} = -2mg \vec{c}$$

$$3) \vec{\Phi}_Q = \Phi_Q \vec{n} = -2mg \vec{n} \quad \vec{n} \text{ normale all' arco AB in } Q.$$

Es. 3

$$1) C(6R \sin \theta, -6R \cos \theta)$$

$$2) T = \frac{1}{2} m \cdot 9R^2 \left( 2 \sin^2 \theta + \frac{11}{6} \right) \dot{\theta}^2$$

$$3) \mathcal{L} = T + U \text{ dove}$$

$$U = \frac{15}{2} mgR \cos \theta + C$$