MR 2011

SOLUZIONI TEMI D'ESAME

$\boxed{18.01.2011}$ – FILA 1 – CIVLT & AMBLT

Es. 1

1) $y_G = \frac{R}{3} \frac{(3\pi - 8)}{(\pi - 2)}$

2) $I_r = \frac{mR^2}{12} \frac{(3\pi - 4)}{(\pi - 2)}$

Es. 2

1) $\mathcal{U} = -\frac{m}{2(4-\pi)} g R \sin\theta - mg\frac{R}{2}\sin\varphi - \frac{1}{2}kR^2\left[\left(\varphi + \frac{\pi}{2} - 2\cos\varphi\right)^2 + (1-\sin\theta)^2\right] + c$

2) $k = \frac{mg}{(4-\pi)R}$

(θ_e, φ_e): $\left(\frac{\pi}{2}, 0\right)$; $\left(\frac{3\pi}{2}; 0\right)$; $\left(\arcsin\frac{1}{4}, 0\right)$; $\left(\pi - \arcsin\frac{1}{4}, 0\right)$

3) (θ_e, φ_e): $\left(\frac{\pi}{2}, -\frac{\pi}{2}\right)$; $\left(-\frac{\pi}{2}, -\frac{\pi}{2}\right)$; $\left(\arcsin\frac{1}{4}, -\frac{\pi}{2}\right)$; $\left(\pi - \arcsin\frac{1}{4}, -\pi/2\right)$

4) $\left(\frac{\pi}{2}, 0\right)$; $\left(-\frac{\pi}{2}, 0\right)$ instabili

$\left(\arcsin\frac{1}{4}, 0\right)$; $\left(\pi - \arcsin\frac{1}{4}, 0\right)$ stabili

Es. 3

1) $\vec{\omega}_S = -\dot{\theta}\,\vec{k}$

2) $T = \frac{1}{2} m \left(\dot{s}^2 + s^2\dot{\theta}^2\right) + \frac{1}{2}\frac{mR^2}{2}\dot{\theta}^2$

3) $\mathcal{L} = T + \mathcal{U}$

$\mathcal{U} = -mg\,s\sin\theta - \frac{mg}{3R} s^2 \sin^2\theta + c$

4) $T + V = E$, $V = -\mathcal{U}$, $E = T_0 + V_0$ vale perché vincoli fissi e bil. e forze conservative.

$\boxed{19.04.2011}$ - FILA 1

Es. 1

1) $x_G = \dfrac{(7\pi + 2)R}{6\pi}$

2) $I_{oy} = \dfrac{(91\pi + 64)mR^2}{24\pi}$

Es. 2

1) $U = -\,mgR\cos\theta - mg\,\xi\cos\theta - mgR\sin\theta - \dfrac{1}{2}\dfrac{mg}{\cancel{3R}}\cdot \cancel{9R^2}^{3R}\cos^2\theta -$

$- \dfrac{1}{2}\dfrac{mg}{3R}(\xi - 3R)^2 + C$

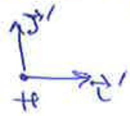
2) (ξ_e, θ_e) dove $\theta_e = \operatorname{arctg}\dfrac{1}{4}$, $\xi_e = 3R(1 - \cos\theta_e)$

3) $\bar{\Phi}_H \equiv \bar{0}$

$\bar{\Phi}_A = mg\left(\dfrac{4}{3} + \cos\theta_e\right)\bar{j}$

4) $\bar{\Phi}_H = mg\cos\theta_e + \dfrac{mg}{3R}(\xi_e - 3R)\,\bar{i}\,' + mg\left(\sin\theta_e + \dfrac{1}{3}\right)\bar{j}\,'$

dove $\vec{i}\,'$ versore della retta AB , $\bar{j}\,' \perp \vec{i}\,'$



5) $\vec{\omega}_D = \left(\dot{\theta} + \dot{\xi}/R\right)\vec{k}$

6) $T = T_{AB} + T_D$

$T_{AB} = \dfrac{1}{2}\cdot\dfrac{m}{3}L^2\sin^2\theta\,\dot{\theta}^2 + \dfrac{1}{2}\cdot\dfrac{m}{\cancel{3}}\cdot\dfrac{1}{\cancel{12}}\cdot\cancel{36}R^2\dot{\theta}^2$

$T_D = \dfrac{1}{2}m\left[\dot{\xi}^2 + R^2\dot{\theta}^2 + \xi^2\dot{\theta}^2 + 9R^2\cos^2\theta\,\dot{\theta}^2 - 6R\xi\cos^2\theta\,\dot{\theta}^2 +\right.$

$\left. 2R\dot{\xi}\dot{\theta} - 6R\sin\theta\cos\theta\,\dot{\xi}\dot{\theta} - 6R^2\sin\theta\cos\theta\,\dot{\theta}^2\right] +$

$+ \dfrac{1}{2}\dfrac{mR^2}{2}\left(\dot{\theta} + \dot{\xi}/R\right)^2$

$\boxed{\text{Mec. 06 11}}$ - FILA 1 - CIVILI & AMBIENT

Es. 1

1) $\beta = \dfrac{11\sqrt{3} - 13}{8}$

2) $I_{Oy} = \dfrac{19}{\cancel{24}6} m \cancel{4} R^2 = \dfrac{19}{6} m R^2$

Es. 2

1) $\mathcal{U} = mg\, L \sin\theta + mg\,(\ell \sin\varphi + 2L \sin\theta) - \frac{1}{2}\, \frac{6mg}{L}\, L^2 \sin^2\theta +$

$- \frac{1}{2} mg \ell \varphi + c$

2) $\left(\frac{\pi}{6}, \frac{\pi}{3}\right); \left(\frac{\pi}{6}, -\frac{\pi}{3}\right); \left(\frac{5}{6}\pi, \frac{\pi}{3}\right); \left(\frac{5}{6}\pi, -\frac{\pi}{3}\right)$

3) $T = T_{AB} + T_{BC}$

$T_{AB} = \frac{1}{2} m L^2 \dot\theta^2 \left(\cos^2\theta + \frac{1}{3}\right)$

$T_{BC} = \frac{1}{2} m \left[(L \sin\theta\, \dot\theta + \ell \sin\varphi\, \dot\varphi)^2 + (2L \cos\theta\, \dot\theta + \ell \cos\varphi\, \dot\varphi)^2 \right]$

$+ \frac{1}{2} \frac{m\ell^2}{3} \dot\varphi^2$

4) $\vec{\Phi}_H(t) = m\,(L \sin\theta\, \ddot\theta + L \cos\theta\, \dot\theta^2 + \ell \sin\varphi\, \ddot\varphi + \ell \cos\varphi\, \dot\varphi^2)\, \vec{i}$

5) $T + V = E$, $E = T_0 + V_0$, $V = -\mathcal{U}$

esiste perché forze conservative e vincoli fissi e bilat.

12.07.2011 – FMA 1

Es. 1

1) $y_G = \frac{(5\sqrt{3}+7)L}{6}$

2) $I_\mu = \frac{1}{24} m L^2 (25 + 21\sqrt{3})$

Es. 2

1) $U = -\frac{1}{4} \frac{mg}{R} s^2 - \frac{1}{2} mgL \sin^2\theta + mg\frac{L}{4}\theta + mg\frac{L}{4R} s + c$

2) (s_e, θ_e) dove $s_e = \frac{L}{2}$

$\theta_e = \frac{\pi}{12}, \frac{13}{12}\pi, \frac{5}{12}\pi, \frac{17}{12}\pi$

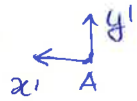
3) $\left(\frac{L}{2}, \frac{\pi}{12}\right)$; $\left(\frac{L}{2}, \frac{13}{12}\pi\right)$ stabili

$\left(\frac{L}{2}, \frac{5}{12}\pi\right)$; $\left(\frac{L}{2}, \frac{17}{12}\pi\right)$ instabili

4) non esistono posizioni di confine di equilibrio

5) $\Phi_O = (0, mg\sin\theta_e)$ da valutare in θ_e.

$\Phi_A = mg\, L/4R\ \vec{i}\,' + \frac{1}{2} mg\, \vec{j}\,'$ dove $A x'y'$ rif. solidale con l'asta OA



6) $T = T_{OA} + T_D$

$T_{OA} = \frac{1}{2} \frac{m}{3} L^2 \dot{\theta}^2$

$T_D = \frac{1}{2} \frac{m}{2} \left[(L-s)^2 \dot{\theta}^2 + \dot{s}^2 + R^2 \dot{\theta}^2 + 2R \dot{s} \dot{\theta} \right] + \frac{1}{2} \frac{mR^2}{4} \left(\dot{\theta} + \frac{\dot{s}}{R} \right)^2$

$\boxed{05.03.2011}$ – FILA 1

<u>Es. 1</u>

1) $x_G = \frac{16}{7} a$

2) $I_{rc} = \frac{17}{2} m a^2$

<u>Es. 2</u>

1) $U = mgL \sin\theta + mgL\,(2\sin\theta + \cos\theta) - \frac{1}{2}\, mg/_L \left[x - L(\sin\theta + \cos\theta)\right]^2 +$

$+ \delta\, mgL\,\theta + c$

2) $\delta = 1 \;\rightarrow\; x_\theta = L \;\Rightarrow\; (x_e, \theta_e) = (L, \frac{\pi}{2})$

3) $T = \frac{1}{2} m \left(\dot{x}^2 + 2L \sin\theta\, \dot{x}\,\dot{\theta} + \frac{4}{3} L^2 \dot{\theta}^2\right) +$

$+ \frac{1}{2} m \left[\dot{x}^2 + 2L\,(\cos\theta + 2\sin\theta)\,\dot{x}\dot{\theta} + \frac{16}{3} L^2 \dot{\theta}^2\right]$

$= m\left(\dot{x}^2 + L\,(\cos\theta + 3\sin\theta)\,\dot{x}\dot{\theta} + \frac{10}{3} L^2 \dot{\theta}^2\right]$

4)

$\ddot{x} + L\sin\theta\,\ddot{\theta} + L\cos\theta\,\dot{\theta}^2 + \ddot{x} + L\cos\theta\,\ddot{\theta} + 2L\sin\theta\,\ddot{\theta} +$

$L\,(2\cos\theta - \sin\theta)\,\dot{\theta}^2 = -g/_L\left[x - L(\sin\theta + \cos\theta)\right]$

$\downarrow$

- $2\ddot{x} + L(3\sin\theta + \cos\theta)\,\ddot{\theta} + L\,(3\cos\theta - \sin\theta)\,\dot{\theta}^2 +$

  $+ g/_L\left[x - L(\sin\theta + \cos\theta)\right] = 0$

$L\,(\sin\theta\,\ddot{x} + \frac{4}{3} L\,\ddot{\theta}) + L\left[(\cos\theta + 2\sin\theta)\ddot{x} + \frac{16}{3} L\,\ddot{\theta}\right] =$

$gL\left[3\cos\theta - \sin\theta + 1\right] + g\left[x - L\,(\sin\theta + \cos\theta)\right]\cdot(\cos\theta - \sin\theta)$

$\downarrow$

- $L\left[(\cos\theta + 3\sin\theta)\ddot{x} + \frac{20}{3} L\ddot{\theta}\right] - gL\,(3\cos\theta - \sin\theta + 1) -$

  $- g\,(\cos\theta - \sin\theta)\left[x - L(\sin\theta + \cos\theta)\right] = 0$

5) $\vec{\Phi}_A(t) = 2mg - mL(\cos\theta\,\ddot{\theta} - \sin\theta\,\dot{\theta}^2) -$

$$- mL\left[(2\cos\theta - \sin\theta)\ddot{\theta} - (2\sin\theta + \cos\theta)\dot{\theta}^2\right]\vec{j}$$

6) $T + V = E$, $V = -\mathcal{U}$, $E = T_0 + V_0$.

$\exists$ perché vincoli fissi e bilat e forze conservative