MR 2012

SOLUZIONI DEI TEMI D'ESAME

17.01.2012 - FILA 1

Es. 1

1) $y_G = \frac{6}{5} R$

2) $I_{Oz} = \frac{43}{3} mR^2$

Es. 2

1) $U = 3mgy + mgx - \frac{1}{2} mg/R (2R-x)^2 - \frac{1}{2} mg/R (y+R)^2 + C$

2) $(x_e = 3R, \; y_e = 2R)$

3) $\bar{\Phi}_H = (-mg, 4mg)$

$\bar{\Phi}_A = -mg\,\bar{\imath}$ + coppia $\vec{M} = 3mgR\,\vec{k}$

4) $3\ddot{y} = 3g - g/R (y+R)$

$\frac{3}{2}\ddot{x} = g - g/R (2R-x)$

5) $\bar{\Phi}_H(t=0) = (\frac{8}{3} mg, 3mg)$

03.04.2012 - FILA 1

Es. 1

1) $I_{11} = \frac{5}{6} m l^2$

$I_{22} = \frac{5}{6} m l^2$

$I_{33} = \frac{5}{3} m l^2$

$I_{12} = -\frac{5}{12} m l^2$

2) $I_C = \frac{5}{24}(4-\sqrt{3}) m l^2$

Es. 2

1) $U = mgR\left[\frac{3}{2}\cos\theta + \frac{\sqrt{3}}{2}\cos\varphi - \frac{3}{4}\cos(\theta-\varphi)\right] + C$

2) (θ_e, φ_e) : $(0,0)$; $(0,\pi)$; $(\pi, 0)$; (π,π)

3) $|\bar{\Phi}_A| = |\bar{\Phi}_B|$ $\bar{\Phi}_A$ e $\bar{\Phi}_B$ hanno la direzione dei raggi CA e CB

~~$(0,0)$ $|\bar{\Phi}_A| = \frac{1-2\sqrt{3}}{4\sqrt{3}} mg$~~

$\Rightarrow \bar{\Phi}_A = \Phi_A \bar{n}$

dove $\Phi_A = \frac{mg}{\sqrt{3}}\left(\cos\varphi - \frac{\sqrt{3}}{2}\cos(\theta-\varphi) - \frac{3}{4}\right)\Big|_{(\theta_e,\varphi_e)}$

da valutare nelle 4 posz. di equilibrio

$(0,0)$; $\bar{\Phi}_A = \frac{mg}{\sqrt{3}}\left(\frac{1}{4} - \frac{\sqrt{3}}{2}\right)\bar{n}$

$(0,\pi)$; $\bar{\Phi}_A = \frac{mg}{\sqrt{3}}\left(-\frac{1}{4} + \frac{\sqrt{3}}{2}\right)\bar{n}$

$(\pi,0)$: $\bar{\Phi}_A = \frac{mg}{\sqrt{3}}\left(\frac{1}{4} + \frac{\sqrt{3}}{2}\right)\bar{n}$

(π,π) $\bar{\Phi}_A = \frac{mg}{\sqrt{3}}\left(-\frac{1}{4} - \frac{\sqrt{3}}{2}\right)\bar{n}$

4) $T = \frac{1}{2} m R^2\left[2\dot{\theta}^2 + \frac{5}{6}\dot{\varphi}^2 + \sqrt{3}\,\dot{\theta}\dot{\varphi}\cos(\theta-\varphi)\right]$

19.06.12 - FILA 1

ES. 1

1) $\gamma = \frac{10\pi}{6-\pi}$

2) $I_{Oy} = \frac{(\pi+4)}{8} m a^2$

ES. 2

1) $U = mg\frac{\sqrt{2}}{2} R \sin\theta - \frac{1}{2}\frac{mg}{R}(x + \sqrt{2}R\cos\theta)^2 - \lambda mg\sqrt{2}R\theta + c$

2) $x_e = -2\lambda\sqrt{2}R$

$\theta_{1e} = \bar{\theta}$, $\theta_{2e} = 2\pi - \bar{\theta}$ dove $\bar{\theta} = \arccos(2\lambda)$ se $\lambda \leq \frac{1}{2}$

$(x_e, \bar{\theta})$; $(x_e, 2\pi - \bar{\theta})$ se $0 < \lambda \leq \frac{1}{2}$

3) $T = \frac{1}{2} m \left[\dot{x}^2 - R\sqrt{2}\sin\theta\,\dot{x}\dot{\theta} + \frac{2}{3}R^2\dot{\theta}^2\right]$

4) $\vec{K}_A = mR\left(\frac{\sqrt{2}}{2}\sin\theta\,\dot{x} - \frac{2}{3}R\dot{\theta}\right)\vec{k}$

5) $\ddot{x} - R\frac{\sqrt{2}}{2}\sin\theta\,\ddot{\theta} - R\frac{\sqrt{2}}{2}\cos\theta\,\dot{\theta}^2 + \frac{g}{R}(x + R\sqrt{2}\cos\theta) = 0$

$\frac{2}{3}R^2\ddot{\theta} - R\frac{\sqrt{2}}{2}\sin\theta\,\ddot{x} - gR\sqrt{2}\left(\frac{1}{2}\cos\theta - \lambda\right) = 0.$

10.07.2012 – FILA 1

Es. 1

1) $l = s\frac{\sqrt{3}}{6}R$

2) $s = \frac{\sqrt{3}}{5}\frac{m}{R^2}$

$l = \frac{1}{10}\frac{m}{R}$

3) $I_{r_{OA}} = \frac{11}{10}mR^2$

Es. 2

1) $U = mg\frac{4R}{3\pi}\cos\theta + mg\xi\sin\theta - \frac{1}{2}\cdot\frac{3\sqrt{2}\pi}{8}\xi^2 + C$

2) (ξ_e, θ_e): $(0,0)$; $(\frac{4R}{3\pi}; \pi/4)$; $(-\frac{4R}{3\pi}, \frac{7}{4}\pi)$

3) $\Phi_H = (0, 2mg)$

4) $(0,0)$ instabile

$(\frac{4R}{3\pi}, \frac{\pi}{4})$; $(-\frac{4R}{3\pi}, \frac{7}{4}\pi)$ stabili

5) $T_{ASTA} = \frac{1}{2}m\left[R^2\dot\theta^2 + \dot\xi^2 + \xi^2\dot\theta^2 + 2R\cos\theta\dot\theta\dot\xi - 2R\xi\sin\theta\dot\theta\right] +$

$+ \frac{1}{2}\cdot\frac{1}{12}m4R^2\dot\theta^2$

$T_{SD} = \frac{1}{2}mR^2\dot\theta^2\left[\left(\frac{4}{3\pi}\sin\theta\right)^2 + \left(1 - \frac{4}{3\pi}\cos\theta\right)^2\right] +$

$+ \frac{1}{2}\left[\frac{mR^2}{2} - m\left(\frac{4R}{3\pi}\right)^2\right]\dot\theta^2$

$= \frac{1}{2}mR^2\dot\theta^2\left[\frac{3}{2} - \frac{8}{3\pi}\cos\theta\right]$

28.08.2019 - FILA 1

Es. 1

1) $G\left(\frac{34}{9}L, \frac{22}{9}L\right)$

2) $I_r = \frac{7}{3} m L^2$

Es. 2

1) $U = -\sqrt{3} mg l \cos\theta + \beta mg 2l \sin\theta - \frac{mg}{2l}(x^2 - 2lx\sin\theta) + C$

2) $\beta = \frac{1}{4} \quad \theta_e = \frac{5}{6}\pi$

3) $\left(\frac{L}{2}, \frac{5}{6}\pi\right)$ stabile

4) $\bar{\Phi}_A = \left(1 - \frac{\sqrt{3}}{2}\right) mg \vec{j}$

$\bar{\Phi}_B = -\frac{1}{4} mg \vec{i}$

$\bar{\Phi}_P = mg\left(1 + \frac{\sqrt{3}}{2}\right)\vec{j}$

5) $T = \frac{1}{2} m \dot{x}^2 + \frac{2}{3}\sqrt{3} m L^2 \dot{\theta}^2$

6) $\omega^2_{1,2} = \frac{(\sqrt{3}+2) \pm \sqrt{7-\sqrt{3}}}{4} g/l$

7) $T + V = E = T_0 + V_0$

$V = -U$

che è tecnicamente vincoli fissi e olonomi e forze conservative.