MR 2013

SOLUZIONI TEMI D'ESAME

15.01.2013 - FILA 1 - CIVLT & AMBLT

Es. 1

1) $l = \dfrac{(\sqrt{3}\pi - 4)}{6\sqrt{3}} sR$

2) $s = \dfrac{6\sqrt{3}\, m}{(3\sqrt{3}\pi - 8) R^2}$

$l = \dfrac{(\sqrt{3}\pi - 4)\, m}{(3\sqrt{3}\pi - 8) R}$

3) $I_{Cz} = \dfrac{(25\sqrt{3}\pi - 88)\, m R^2}{6(3\sqrt{3}\pi - 8)}$

Es. 2

1) $\mathcal{U} = -\frac{1}{2} k x^2 - mgx + c$

2) $x_e = -mg/k$

3) STABILE

4) $T = \frac{5}{4} m \dot{x}^2$

5) $\vec{\mathcal{E}}_A = -2mL\dot{x}\vec{k}$

6) $\omega^2 = \frac{2}{5} k/m$

7) $V = \frac{k}{2} x^2 + mgx$

x_e è un centro.

26.03.2013 - FILA 1 - CNLT & AMBLT

Es. 1

1) $y_G = \frac{2(17+9\pi)}{(16+9\pi)} L$

2) $I_{Hx'} = \frac{1024+243\pi}{48(16+9\pi)} ml^2$

$I_{Hy} = \frac{64+243\pi}{48(16+9\pi)} ml^2$

$I_{Hz} = \frac{544+243\pi}{24(16+9\pi)} ml^2$

3) $I_K = \frac{544+243\pi}{48(16+9\pi)} mR^2$

Es. 2

1) $U = -\frac{1}{2} k s^2 + mg(l-s)\cos\theta - \frac{1}{4} mg(2l-s)\cos\theta + C$

2) (s_e, θ_e): $(\frac{3}{4}\frac{l}{\beta}, \pi)$ $\exists$ se $\beta > \frac{3}{8}$

$(\frac{2}{3}l, \bar{\theta})$ e $(\frac{2}{3}l, 2\pi - \bar{\theta})$ dove $\bar{\theta} = \arccos(-\frac{8}{9}\beta)$

esistono se $0 < \beta \leq \frac{9}{8}$

3) $(0,0)$ $\forall \beta > 0$

$(2l, \pi)$ se $\beta \leq \frac{3}{8}$

4) $(\frac{3}{4}\frac{l}{\beta}, \pi)$ stabile se $\beta < \frac{9}{8}$

$(\frac{2}{3}l, \bar{\theta})$ e $(\frac{2}{3}l, 2\pi - \bar{\theta})$ dove entrambi sono instabili

$\beta = 9/8$ pto di bif. INSTABILE

5) $\bar{\Phi}_D = \Phi_D \bar{n}$ dove $\bar{n} \perp$ ad AB

$\Phi_D(\theta_e = \pi) = 0$

$\Phi_D(\theta_e = \bar{\theta}) = \frac{1}{12} mg \sqrt{81 - 64\beta^2}$

$\Phi_D(\theta_e = 2\pi - \bar{\theta}) = -\frac{1}{12} mg \sqrt{81 - 64\beta^2}$

6) $T = \frac{1}{2} m \left[\dot{s}^2 + (l-s)^2 \dot{\theta}^2 + \frac{1}{3} \ell^2 \dot{\theta}^2 \right]$

7) $\omega_1^2 = \frac{1}{2} g/\ell$

$\omega_2^2 = \frac{54}{7} g/\ell$

11.06.2013 - FILA 1 - CIVLT & AMBLT

Es. 1

1) $l = \frac{RS}{9\sqrt{3}}$

2) $S = \frac{18}{11\sqrt{3}}\, m/R^2 = \frac{6}{11}\sqrt{3}\, m/R^2$

$l = \frac{2}{33}\, m/R$

3) $I_{Ox} = \frac{93}{88} m R^2$

Es. 2

1) $U = mgR\cos\theta - \frac{1}{2}\, mg/R\,(x^2 - 2Rx\sin\theta) + Fx + C$

2) $x_e = R \Rightarrow \theta_e = \pi/4$, $F = mg\left(1 - \frac{\sqrt{2}}{2}\right)$

3) $(R, \pi/4)$ stabile

4) $\ddot{x} + g/R\,(x - R\sin\theta) - F = 0$

$R^2\ddot{\theta} + gR\sin\theta - gx\cos\theta = 0$

5) $\bar{\Phi}_D = \frac{3(l+2R)}{4e}\, mg\bar{J}$

$\bar{\Phi}_C = \frac{3(l-2R)}{4e}\, mg\bar{J}$

02.07.2013 - FILA 1

Es. 1

1) $y_G = \frac{4R}{15}\left(2+\frac{1}{\pi}\right)$

2) $I_x = \frac{29}{24} m R^2$

Es. 2

1) Utilizzando la formula fondamentale della cinematica dei sistemi rigidi

2) $U = -4mgR\cos\theta - 2mgR\cos^2\theta + 6mgR\sin^2\theta + c$

3) $\theta_e = 0, \pi, \frac{\pi}{3}, \frac{5}{3}\pi$

4) $T = mR^2\left(\frac{8}{3}+\cos^2\theta\right)\dot\theta^2$

5) $\vec{k}_G = mR^2\left(\frac{4}{3}+4\sin^2\theta-3\cos\theta\right)\dot\theta\,\vec{k}$

6) $\theta_e = 0$ $\quad \bar\Phi_G \equiv \bar 0$

$\bar\Phi_C = (0, 5mg)$

$\bar\Phi_B = (0, -4mg)$

$\theta_e = \pi$ $\quad \bar\Phi_G \equiv \bar 0$

$\bar\Phi_C = (0, mg)$

$\bar\Phi_B \equiv \bar 0$

$\theta_e = \pm\frac{\pi}{3}$ $\quad \bar\Phi_G = \pm 3\sqrt{3}\,mg\,\vec{i}$

$\bar\Phi_C = (\pm 3\sqrt{3}\,mg, 4mg)$

$\bar\Phi_B = (\pm 3\sqrt{3}\,mg, -3mg)$

02.09.2013 - FILA 1 - CIVLT & AMBLT

Es. 1

1) $G\left(\frac{23}{24}R, \frac{17}{24}\sqrt{3}R\right)$

2) $I_{11} = \frac{21}{4} m R^2$

$I_{22} = \frac{3}{4} m R^2$

$I_{33} = 6 m R^2$

$I_{12} = -\frac{3}{4}\sqrt{3} m R^2$

3) $I_{\pi} = \frac{19}{6} m R^2$

Es. 2

1) $\theta \in [\arcsin\frac{1}{4}, \pi - \arcsin\frac{1}{4}]$

2) $\vec{\omega}_{OA} = -\dot{\theta}\vec{k}$

$\vec{\omega}_{\delta} = -\frac{\dot{\theta}}{\sin^2\theta}\vec{k}$

3) $U = -mgR(4\cos\theta + \operatorname{cotg}\theta + \beta(1-\cos\theta)^2) + c$

4) $\beta = \frac{5}{2}$

5) $\theta_e = \frac{\pi}{2}$ stabile

6) $T = \frac{1}{2} m R^2 \dot{\theta}^2 \left(\frac{32}{3} + \frac{3}{2\sin^4\theta}\right)$

7) $\omega^2 = \frac{30}{43} g/R$