MR 2014

SOLUZIONI TEMI D'ESAME

15.01.2014 – FILA 1 – CIVLT & AMBLT

Es 1

1) $G\left(\frac{17}{24}\sqrt{3}R, \frac{23}{24}R\right)$

2) $I_{11} = \frac{3}{4}mR^2$

$I_{22} = \frac{21}{4}mR^2$

$I_{33} = 6mR^2$

$I_{12} = -\frac{3}{4}\sqrt{3}mR^2$

3) $I_{Ox} = \frac{37}{24}mR^2$

Es 2

1) $U = -\frac{1}{2}mg/R(4R-x)^2 - \frac{1}{2}mg/R\left[(x+R\cos\theta)^2 + (R\sin\theta - 2R)^2\right]$

$+ H\theta + c$

2) $M = -2mgR \qquad x_e = 2R$

3) $(2R, \pi/2)$ stabile

4) $T = \frac{1}{2}m\dot{x}^2 + \frac{1}{4}mR^2\dot{\theta}^2$

5) $\begin{cases} \ddot{x} + 2g/R\,x = -g(\cos\theta - 1) \\ \frac{mR^2}{2}\ddot{\theta} = mg(x\sin\theta + 2R\cos\theta) + M \end{cases}$

6) $\bar{\Phi}_G(t) = mg\,(\sin\Theta - 1)\,\bar{J}$

7) $\omega^2_{1,2} = (3 \pm \sqrt{5})\,g/R$

15.04.14 - FILA 1 - AMBLT & CIVLT

ES. 1

1) $G\left(\frac{17}{12}R, \frac{7}{12}\sqrt{3}R\right)$

2) $I_{Oz} = \frac{29}{3}\mu R^2$

3) $I_{xe} = \frac{33}{8}\mu R^2$

ES. 2

1) $a_l = -mg\frac{L}{6}\sin\theta - \frac{1}{2}mg/L\left(x^2 - \frac{4}{3}L\cos\theta\, x\right) + \beta mg x + M\theta + c$

2) $M = -mg\frac{L}{6}$, $x_e = \left(\beta - \frac{2}{5}\right)L$

3) (x_e, π) stabile se $\beta < \frac{2}{3}$

4) $T = \frac{1}{2}\mu\dot{x}^2 + \frac{1}{18}\mu L^2\dot{\theta}^2$

5) $\ddot{x} + 8/L\, x - \frac{2}{3}g\cos\theta - \beta g = 0$

$\frac{mL^2}{9}\ddot{\theta} + mg\frac{L}{6}\cos\theta + \frac{2}{3}mg\, x\sin\theta - M = 0$

6) $\bar{\Phi}_p(t) = mg\left(1 - \frac{2}{3}\sin\theta\right)\bar{j}$

17.06.2016 | - FILA 1 - CIVLT & AMBLT

Es. 1

1) $G(2R, 0)$

2) $I_z = \frac{22}{3} mR^2$

3) $I_x = \frac{3}{2} mR^2$

4) $I_y = \frac{35}{6} mR^2$

Es. 2

1) $\vec{\omega}_D = \frac{2L}{R} \cos\theta \, \dot{\theta} \, \vec{k}$

2) $U = mg\left(\frac{4}{6}\theta - \frac{1}{2}\sin\theta\right) + C$

3) $\begin{cases} \theta_1 = \arccos\frac{1}{3} & \text{INSTABILE} \\ \theta_2 = 2\pi - \arccos\frac{1}{3} & \text{STABILE} \end{cases}$ ← 4)

5) $\vec{\Phi}_C = mg\left(\frac{4/6 - 2\cos\theta_e}{\sin\theta_e}\right)\vec{i}$

$\vec{\Phi}_G = -mg\left(\frac{4/6 - 2\cos\theta_e}{\sin\theta_e}\right)\vec{i} + 4mg\,\vec{j}$

6) $\vec{\Phi}_A = -mg\left(\frac{4/6 - 2\cos\theta_e}{\sin\theta_e}\right)\vec{i} + mg\,\vec{j}$

$\vec{\Phi}_B = mg\left(\frac{4/6 - 2\cos\theta_e}{\sin\theta_e}\right)\vec{i} - 2mg\,\vec{j}$

$\frac{\frac{4}{6} - 2\cos\theta_e}{\sin\theta_e} = \pm\frac{3\sqrt{2}}{8}$

7) $T = \frac{1}{2}\left(1 + 8\cos^2\theta\right) mL^2 \dot{\theta}^2$

02.07.2014 - FILA 1 - CIVLT & AMBLT

Es 1

1) $G_1 \left(\frac{7\sqrt{3}}{8} R, \frac{7}{8} R \right)$

2) $I_{11} = \frac{5}{6} m R^2$

$I_{22} = \frac{5}{2} m R^2$

$I_{33} = \frac{10}{3} m R^2$

$I_{12} = -\frac{5}{12} \sqrt{3} m R^2$

3) $I_{G_2} = \frac{74}{81} m R^2$

Es. 2

1) $\bar{\omega}_3 = -3\dot{\theta}\,\vec{k}$

2) $U = -\frac{9}{2} k R^2 \sin^2\theta - 6FR\cos\theta + C$

3) $\theta_1 = 0$

$\begin{cases} \theta_2 = \bar{\theta} \\ \theta_3 = 2\pi - \bar{\theta} \end{cases}$ dove $\bar{\theta} = \arccos \frac{2F}{3kR}$ esiste se $F \le \frac{3}{2} kR$

4) θ_1 STABILE se $F < \frac{3}{2} kR$

θ_2, θ_3 INSTABILI dove esistono

$F = \frac{3}{2} kR$ pto di biforcazione instabile

5) $T = \frac{3}{4} m R^2 (11 + 12 \sin^2\theta)\,\dot{\theta}^2$

6) $\frac{3}{2} m R^2 (11 + 12\sin^2\theta)\,\ddot{\theta} + 18 m R^2 \sin\theta\cos\theta\,\dot{\theta}^2 +$

$+ 9 k R^2 \sin\theta\cos\theta - 6FR\sin\theta = 0.$

25.08.2014 - FILA 1 - CNLT & AMBLT

Es 1

1) $G\left(\frac{28\sqrt{3}-3}{48}R;\ \frac{3\sqrt{3}+22}{48}R\right)$

2) $I_{11} = \frac{19}{12}mR^2$

$I_{22} = \frac{73}{12}mR^2$

$I_{33} = \frac{92}{12}mR^2 = \frac{23}{3}mR^2$

$I_{12} = -\frac{17}{12}\sqrt{3}mR^2$

3) $I_r = \frac{17}{6}mR^2$

Es. 2

1) $\theta \in [0, 2\pi)$

$s \in [-2R, 2R]$

2) $a = -mg\,s\cos\theta + \frac{1}{4}mgR\theta + C$

3) (s_e, θ_e): $\left(\frac{1}{4}R, \pi/2\right)$; $\left(-\frac{1}{4}R, -\pi/2\right)$

4) $\left(2R, \arcsin\frac{1}{8}\right)$

$\left(-2R, \arcsin\frac{1}{8}+\pi\right)$

5) p. di eq. ordinarie instabili

6) $T = \frac{1}{2}m\left[\dot{s}^2 + \left(s^2 + \frac{4}{3}R^2\right)\dot{\theta}^2\right]$

7) $\ddot{s} - s\dot{\theta}^2 - g\cos\theta = 0$

$\left(\frac{4}{3}R^2 + s^2\right)\ddot{\theta} + 2s\dot{s}\dot{\theta} + g\,s\,\sin\theta - \frac{1}{4}gR = 0$

8) $\vec{\Phi}_o(t) = m\ddot{x}_G \vec{\imath} + (m\ddot{y}_G + 2mg)\vec{\jmath}$

dove

$\ddot{x}_G = \sin\theta\,\ddot{s} + 2\dot{s}\cos\theta\,\dot{\theta} + s\cos\theta\,\ddot{\theta} - s\sin\theta\,\dot{\theta}^2$

$\ddot{y}_G = -\ddot{s}\cos\theta + 2\sin\theta\,\dot{s}\dot{\theta} + s\cos\theta\,\dot{\theta}^2 + s\sin\theta\,\ddot{\theta}$