SOLUZIONI TEMI D'ESAME

MR 2015

$\boxed{13.01.2015}$ - CIV & AMB - FILA 1

Es. 1

1) $G\left(\frac{9}{20}, \frac{9}{20}\right)$

2) $I_{11} = I_{22} = \frac{9}{35} m$

$I_{33} = 2 I_{11}$

$I_{12} = -\frac{1}{4} m$

3) $I_r = \frac{m}{140}$

Es. 2

1) $U = mg\frac{R}{2}\theta - \frac{1}{2} mgR\,\theta^2 + \frac{mgR}{12}\theta^3 + c$

2) $\theta_e = 2 \pm \sqrt{2} \begin{cases} + & \text{instabile} \\ - & \text{stabile} \end{cases}$ ← 3)

4) $\bar{\Phi}_c = -\frac{mg}{2}(\sqrt{3}\theta_e + 1)\,\bar{\imath} + mg(1 + \sqrt{3} - \theta_e)\,\bar{\jmath}$

da valutare in θ_e

5) $T = \frac{3}{4} m R^2 \dot{\theta}^2$

6) $\ddot{\theta} + \frac{1}{6} \frac{g}{R}(\theta^2 - 4\theta + 2) = 0$

7) $\omega_0^2 = \frac{g}{\cancel{3}R}(\cancelto{8}{24} - \cancelto{4}{12}\sqrt{3}) = 4\, g/R\, (2 - \sqrt{3})$

$\boxed{31.03.15}$ – FILA 1 – CIVILI & AMBLT

ES. 1

1) $x_G = \frac{(7+5\sqrt{3})L}{12}$

2) $I_O = \frac{mL^2}{24}\,\mathrm{diag}\left[(1+\sqrt{3});\ 2(11+9\sqrt{3});\ (23+19\sqrt{3})\right]$

3) $I_u = \frac{(25+21\sqrt{3})mL^2}{96}$

ES. 2

1) $\vec{\omega}_D = \left(\dot{\theta} - \frac{\dot{s}}{R}\right)\vec{k}$

2) $U = -mg\frac{L}{4R}s \overset{+}{(-)} mg\frac{L}{4}\theta - \frac{mg}{2}L\sin^2\theta - \beta\frac{mg}{4R}(L-s)^2 + c$

3) $s_e = L\left(1 - \frac{1}{2\beta}\right)$ $\exists$ se $\beta > \frac{1}{2}$

$\theta_{ie} = \overset{(1)}{\frac{\pi}{12}};\ \overset{(2)}{\frac{13}{12}\pi};\ \overset{(3)}{\frac{5}{12}\pi};\ \overset{(4)}{\frac{17}{12}\pi}$

4 posizioni (s_e, θ_{ie}) $i = 1, 2, 3, 4$ se $\beta > \frac{1}{2}$

4) $\left(s_e, \frac{\pi}{12}\right);\ \left(s_e, \frac{13}{12}\pi\right)$ stabili se $\beta > \frac{1}{2}$.

5) 4 posiz. di confine di equilibrio

$(0, \theta_{ie})$ $i = 1, 2, 3, 4$ se $\beta \leq \frac{1}{2}$

6) $T = \frac{1}{2}\left[\cancel{4}\frac{m}{3}L^2\dot{\theta}^2 + \beta\frac{m}{2}s^2\dot{\theta}^2 + \frac{3}{4}m\beta(\dot{s} - R\dot{\theta})^2\right]$

16. 06. 2015 - FILA 1 - CIVILI & AMBIT

Es. 1

1) $G\left(\frac{5}{3}L, \frac{7}{3}L\right)$

2) $I_{11} = \frac{20}{3}mL^2$

$I_{22} = 4mL^2$

$I_{33} = \frac{32}{3}mL^2$

$I_{12} = -\frac{13}{3}mL^2$

3) $I_r = 4mL^2$

Es. 2

1) $U = -mg\frac{L}{3}\sin\theta + \beta mg\left(x + \frac{L}{3}\cos\theta\right) - \frac{1}{2}\frac{mg}{L}x^2 + \frac{2}{9}mgL\sin\theta + c$

2) $\theta_{1e} = \bar\theta$; $\theta_{2e} = \bar\theta + \pi$ dove $\bar\theta = \operatorname{arctg}\left(-\frac{1}{3\beta}\right)$

$x_e = \beta L$

$\Rightarrow (\beta L, \bar\theta)$; $(\beta L, \bar\theta + \pi)$

se $\beta > 0$ $x_e \in Ox^+$ $\theta_{1e} \in (\pi/2, \pi)$

3) (θ_{1e}, x_e) instabile; (x_e, θ_{2e}) stabile

4) $T = \frac{1}{2}m\left(\dot x^2 - \frac{2}{3}L\sin\theta\,\dot x\dot\theta + \frac{L^2}{6}\dot\theta^2\right)$

5) $\ddot x - \frac{L}{3}\sin\theta\,\ddot\theta - \frac{L}{3}\cos\theta\,\dot\theta^2 - \beta g + \frac{g}{L}x = 0$

$\frac{L}{6}\ddot\theta - \frac{1}{3}\sin\theta\,\ddot x + \frac{g}{9}(\cos\theta + 3\beta\sin\theta) = 0$

6) $\Phi_A(t=0) = \frac{7}{9}mg\vec{j}$

$\boxed{07.07.2015}$ - FILA 1 - CIVLTB AMBLT

ES. 1

1) $y_G = \frac{19}{18} a$

2) $I_O = ma^2 \operatorname{diag}\left(\frac{53}{6}, \frac{5}{12}, \frac{37}{4}\right)$

3) $I_r = \frac{323}{48} ma^2$

ES. 2

1) $U = mg\left(5L \sin\theta - s\cos\theta - \frac{\alpha}{2L} s^2\right) + C$

2) $(0, \frac{\pi}{2})$; $(0, \frac{3}{2}\pi)$ $\exists\ \forall \alpha > 0$

$(\bar{s}_0, \bar{\theta}_0)$; $(-\bar{s}_0, \pi - \bar{\theta})$ $\exists$ se $0 < \alpha \leq \frac{1}{5}$

dove $\bar{\theta} = \arcsin 5\alpha$

$\bar{s} = -\frac{L}{\alpha} \cos\bar{\theta}$

3) $(0, \frac{\pi}{2})$ stabile se $\alpha > \frac{1}{5}$

$(0, \frac{3}{2}\pi)$ instabile $\forall \alpha$

$(\bar{s}, \bar{\theta})$; $(-\bar{s}, \pi - \bar{\theta})$ stabili se $\alpha < \frac{1}{5}$ $\left(0 < \alpha < \frac{1}{5}\right)$ dove esistono

$\alpha = \frac{1}{5}$ pto di biforcazione stabile

4) $(-L, \theta_1)$ $\exists$ se $\alpha \leq \cos(\operatorname{arctg} 5)$

$\theta_1 = \operatorname{arctg} 5$

(L, θ_2) $\exists$ se $\alpha \leq -\cos(\operatorname{arctg}(-5))$

$\theta_2 = \operatorname{arctg}(-5)$

5) $T = \frac{1}{2} m \left[\dot{s}^2 - 4L \dot{s}\dot{\theta} + \left(\frac{29}{3} L^2 + s^2\right) \dot{\theta}^2\right]$

$\boxed{01.08.2015}$ – FILA 1 – CIVLT & AMBLT

Es. 1

1) $y_G = \frac{\sqrt{2}}{6} h$

2) $I_O = mL^2 \operatorname{diag}\left(\frac{1}{4}, \frac{5}{12}, \frac{2}{3}\right)$

3) $I_{Gr} = \frac{11}{36} mL^2$

Es. 2

1) $U = 2mg\, s \sin\alpha - \frac{1}{2} \frac{mg}{R} s^2 + mgR\cos\varphi + mg\, s\cos\alpha$

$+ mgR \sin\varphi + C$

2) $s_e = 2R\sin\alpha + R\cos\alpha$

$\varphi_{1e} = \pi/4 \qquad \varphi_{2e} = \frac{5}{4}\pi$

3) $(2R\sin\alpha, \pi/4)$ ↓ INSTABILE $\quad (2R\sin\alpha, \frac{5}{4}\pi)$ ↓ STABILE

4) $\bar{\Phi}_C = -mg\cos\alpha\, \vec{i} + mg\,(1 + 2\cos\alpha - \sin\alpha)\, \vec{j}$

5) $\bar{\Phi}_P = \Phi_P\, \vec{n}^{\circ} \qquad \vec{n}^{\circ} \parallel$ al raggio OP

$|\bar{\Phi}_P| = \sqrt{2}\, mg$

6) $T = \frac{3}{4} m\dot{s}^2 + \frac{1}{2} m\left[\dot{s}^2 + R^2\dot{\varphi}^2 - 2R\dot{s}\dot{\varphi}\cos(\varphi+\alpha)\right]$

$= \frac{1}{2} m\left[\frac{5}{2}\dot{s}^2 + R^2\dot{\varphi}^2 - 2R\dot{s}\dot{\varphi}\cos(\varphi+\alpha)\right]$