# MR 2016

## SOLUZIONI TEMI D'ESAME

$\boxed{12.01.2016}$ – FILA 1

### Es. 1

1) $G\left( R\,\dfrac{(25-3\pi)}{3(6-\pi)} \,;\, R\,\dfrac{(32-3\pi)}{6(6-\pi)} \right)$

2) $I_{11} = \dfrac{(32-3\pi)}{4(6-\pi)}\, m R^2$

$I_{22} = \dfrac{9\,(8-\pi)}{4\,(6-\pi)}\, m R^2$

$I_{33} = \dfrac{(25-3\pi)}{(6-\pi)}\, m R^2$

$I_{12} = -\dfrac{7}{(6-\pi)}\, m R^2$

### Es. 2

1) $\mathcal{U} = 3mgy + mgR\cos\theta - \frac{2}{3}\,{}^{mg}/_{R}\,(y^2 - 6Ry\cos\theta) - \lambda mgR\cos\theta + c$

2) $\left(\frac{21}{4}R, 0\right)\ \exists\ \forall \lambda \in \mathbb{R}$ (*)

$\left(\left(\frac{\lambda-1}{4}\right), \pm\bar{\theta}\right)\ \exists$ se $1 < \lambda \leq 22$ (**)

dove $\bar{\theta} = \arccos\left(\frac{\lambda-10}{12}\right)$

3) (*) stabile se $\lambda < 22$

(**) dove esistono sono instabili

$\lambda = 22$ pto di biforcazione instabile

4)

per $\theta = 0$ $\quad \Phi_P = 0 \quad ; \quad \bar{\Phi}_O = -4mg\,\bar{j}$

per $\theta = \bar{\theta}$ $\quad \bar{\Phi}_P = -4mg \sin\bar{\theta}\,\bar{i} \quad ; \quad \bar{\Phi}_O = (4mg \sin\bar{\theta}, -4mg)$

per $\theta = -\bar{\theta}$ $\quad \bar{\Phi}_P = 4mg \sin\bar{\theta}\,\bar{i} \quad ; \quad \bar{\Phi}_O = (-4mg \sin\bar{\theta}, -4mg)$

5) $T = \frac{1}{2}\left(3m\dot{y}^2 + \frac{7}{2} mR^2\dot{\theta}^2\right)$

6) $\omega_1^2 = \frac{4}{9}\, g/R$

$\omega_2^2 = \frac{30}{7}\, g/R$

$\boxed{22.03.2016}$ - FILA 1 - CIV & AMB

ES. 1

1) $x_G = \frac{(44+7\sqrt{2})R}{44}$

$y_G = \frac{7}{44}\sqrt{2}\,R$

2) $I_M = \frac{205}{176} mR^2$

ES. 2

1) $0 \leq \theta \leq \pi$

2) $\vec{\omega}_{AB} = \frac{\dot{\theta}}{2}\vec{k}$

3) $\mathcal{U} = -mg\frac{L}{2}\sin\theta - mgL\left(\sin\theta - \cos\frac{\theta}{2}\right) - 2mgL\sin\theta + c$

$= -mg\frac{7}{2}L\sin\theta + mgL\cos\frac{\theta}{2} + c$

4) $\theta = \frac{\pi}{3}$ unica stabile $\leftarrow 5)$

6) $\bar{\Phi}_C = \frac{1}{2}mg\,\vec{n}$ dove $\vec{n}$ versore normale ad $\bar{AB}$

$\bar{\Phi}_0 = -\frac{\sqrt{3}}{4}mg\,\bar{\imath} - \frac{1}{4}mg\,\bar{\jmath}$

7) $T = \frac{1}{2}\frac{mL^2}{3}\dot{\theta}^2\left[5 - 3\sin\frac{\theta}{2}\right]$

8) $\omega^2 = \frac{9\sqrt{3}}{28}g/L$

$\boxed{14.06.2016}$ – FILA 1 – CN 8 AMB

## Es. 1

1) $x_G = \frac{4}{3} L$

$y_G = \frac{(2\sqrt{3}-1)L}{3}$

2) $I_{11} = \frac{28}{3} m L^2$

$I_{22} = \frac{20}{3} m L^2$

$I_{33} = 16 m L^2$

$I_{12} = (1 - \frac{10}{3}\sqrt{3}) m L^2$

3) $I_r = (7 + \frac{10}{3}\sqrt{3}) m L^2$

## Es. 2

1) $\theta \in [\arcsin \frac{1}{4}, \pi/4]$

2) $\vec{\omega}_{AB} = -\dot{\theta}\vec{k}$

$\vec{\omega}_{g} = -\frac{1}{\sin^2\theta}\dot{\theta}\vec{k}$

3) $\mathcal{U} = mgR(3\cot\theta - 2\cos\theta) - 3mgR\cot\theta - mgR\theta + c$

$= -mgR(2\cos\theta + \theta) + c$

4) $\theta_e = \pi/6$

5) $\bar{\Phi}_0 = \frac{1}{2} mg \vec{n}$ $\quad \bar{n}$ perpendicolare ad AB

$\bar{\Phi}_C = (3 - \frac{\sqrt{3}}{4}) mg \bar{i} + \frac{1}{4} mg \bar{j}$

$\Phi_B = \frac{\sqrt{3}}{4} mg \bar{i} + \frac{3}{4} mg \bar{j}$

6) $T = 2mR^2\dot{\theta}^2 [\frac{4}{3} - \frac{1}{\sin\theta} + \frac{1}{\sin^4\theta}]$

$\boxed{06.07.2016}$ – FILA 1 – CN & AMB

## Es. 1 (raggio $\overline{OE} = 4R$)

1) $G \in Ox$

$$x_G = \frac{(64 - 21\pi)}{17\pi} R$$

2) $$I_{11} = \frac{mR^2}{17}\left(\frac{45}{4} - \frac{48}{\pi}\sqrt{3} + 32\right)$$

$$I_{22} = mR^2\left(\frac{263}{34} - \frac{45}{4} + \frac{48}{\pi}\sqrt{3} - 32\right)$$

$$I_{33} = \frac{263}{34} m R^2$$

## Es. 2

1) $U = 2mgR\cos\theta\left[(\alpha+1) - 4\cos\theta\right] + C$

2) $\theta = 0$, $\theta = \pi$ indip da $\alpha$

$\theta = \pm\bar{\theta}$ dove $\bar{\theta} = \arccos\left(\frac{\alpha+1}{8}\right)$ esistono se $\alpha \in (0, 7]$

3) $\theta = 0$ stabile se $\alpha > 7$

$\theta = \pi$ instabile $\forall \alpha > 0$

$\theta = \pm\bar{\theta}$ stabili se $0 < \alpha < 7$

$\alpha = 7$ pto di biforcazione stabile

4) $\bar{\Phi}_0 = -mg\,\bar{i} + (7-\alpha)mg\,\bar{j}$ in $\theta = 0$

$\bar{\Phi}_0 = mg\,\bar{i} - (9+\alpha)mg\,\bar{j}$ in $\theta = \pi$

$\bar{\Phi}_0 = -mg\left(\frac{\alpha+1}{8}\right)\bar{i} + mg\sin\bar{\theta}\,\bar{j}$ in $\bar{\theta}$

$\bar{\Phi}_0 = -mg\left(\frac{\alpha+1}{8}\right)\bar{i} - mg\sin\bar{\theta}\,\bar{j}$ in $-\bar{\theta}$

5) $$T = \frac{1}{2}\left(\frac{32\alpha + 27}{6}\right) mR^2\dot{\theta}^2$$

$\boxed{30.08.2016}$ - FILA 1 - CIN & ANB

## Es. 1

1) $G\left(0, \frac{2}{3}\sqrt{3}L\right)$

2) $I_{11} = \frac{35}{18} mL^2$

$I_{22} = \frac{1}{2} mL^2$

$I_{33} = \frac{22}{9} mL^2$

3) $I_r = \frac{3}{2} mL^2$

## Es. 2

1) $\vec{\omega}_D = -\frac{(R-r)}{r}\dot{\theta}\,\vec{k}$

2) $U = mg(R-r)\left[(\alpha+1)\cos\theta + \alpha\theta\right] + C$

3) $\theta_{1e} = \bar{\theta}$

$\theta_{2e} = \pi - \bar{\theta}$ dove $\bar{\theta} = \arcsin\left(\frac{\alpha}{\alpha+1}\right)$ che $\exists\ \forall \alpha > 0$

4) $\theta_{1e}$ stabile, $\theta_{2e}$ instabile

5) $\vec{\Phi}_C = \alpha mg\left(\frac{R-r}{R}\right)\sin\theta_e\,\vec{i} + mg\left[(\alpha+1) - \alpha\left(\frac{R-r}{R}\right)\cos\theta_e\right]\vec{j}$

6) $T = \frac{3}{4} m (R-r)^2\dot{\theta}^2$

7) $\vec{V}_G = (R-r)\dot{\theta}^*\vec{t}$ $\quad\vec{t}$ versore tg alla circonferenza descritta da G.

$$(\dot{\theta}^*)^2 = \frac{4}{3}\frac{g}{(R-r)}\left[(\alpha+1)(\cos\theta - 1) + \alpha\theta\right]\Big|_{\theta=\frac{3}{2}\pi,\ \alpha=1}$$

$$= \frac{4}{3}\frac{g}{(R-r)}\left(\frac{3}{2}\pi - 2\right) > 0.$$