MR 2017.

SOLUZIONI ~~TEMI~~ D' ESAME.

$\boxed{10.01.2017}$ - FILA 1 - CIV 8 AMB

Es. 1

1) $G\left(0, \frac{(64\sqrt{3}-15\pi)}{100\pi} R\right)$

2) $I_{11} = \left(\frac{385}{64} + \frac{3\sqrt{3}}{\pi}\right) mR^2$

$I_{22} = \left(\frac{301}{64} - \frac{3\sqrt{3}}{\pi}\right) mR^2$

$I_{33} = \frac{343}{32} mR^2$

3) $I_r = \frac{363}{64} mR^2$

Es. 2

1) $\theta \in \left[\frac{\pi}{6}, \frac{5\pi}{6}\right] = I$

2) $\bar{\omega}_{OA} = \dot{\theta}\vec{k}$

$\bar{\omega}_{\mathcal{D}} = \frac{\dot{\theta}}{\sin^2\theta} \vec{k}$

3) $\mathcal{U} = -mgL\sin\theta - mg\frac{L}{\alpha}\cos\theta + c$

4) $\theta_1 = \operatorname{arctg}\alpha \in \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$ acc. $\to$ INSTABILE $\leftarrow$ 5)

$\theta_2 = \theta_1 + \pi \notin I$

6) $V = mgL\sin\theta + mg\frac{L}{\alpha}\cos\theta \quad \theta \in \left[\frac{\pi}{6}, \frac{5}{6}\pi\right]$

θ_1 max $\to$ sella.

7) esterne $\bar{\Phi}_O$, $\bar{\Phi}_C$

interne $\bar{\Phi}_O'$

$$\overline{\Phi}_O = mg \sin^2\theta_1 \cos\theta_1 \vec{\imath} + mg\,(1 - \sin\theta_1 \cos^2\theta_1)\,\bar{J}$$

$$\overline{\Phi}_C = -mg \sin^2\theta_1 \cos\theta_1 \vec{\imath} + mg\,(1 + \sin\theta_1 \cos^2\theta_1)\,\bar{J}$$

(normale esterna)

$\overline{\Phi}_{O'} = mg \sin\theta_1 \cos\theta_1 \vec{n}$ dove $\vec{n}$ è il versore della retta $\perp$ ad OA passante per O'.

8) $$T = \frac{2}{3} m l^2 \dot\theta^2 + \frac{3}{4} m l^2 \frac{\dot\theta^2}{\sin^4\theta}$$

$$= m l^2 \dot\theta^2 \left(\frac{2}{3} + \frac{3}{4 \sin^4\theta} \right)$$

11.04.2017 CV & AMB

Es. 1

1) $\alpha = \frac{4}{3}$

2) $I_O = \frac{2}{9} m R^2 \operatorname{diag}\left(\frac{29}{3}, \frac{35}{12}, \frac{151}{12}\right)$

Es. 2

1) $U = -\frac{1}{2} mgl \cos\theta + \frac{1}{2} mgl \cos\varphi - \frac{1}{2} \frac{mgl}{\lambda} \sin^2\theta + c$

2) (θ_e, φ_e): $(0,0)$; $(0,\pi)$; $(\pi,0)$; (π,π) indip da λ.

$(\bar\theta,0)$; $(\bar\theta,\pi)$; $(-\bar\theta,0)$; $(-\bar\theta,\pi)$

dove $\bar\theta = \arccos\frac{\lambda}{2}$ che $\exists$ se $0<\lambda\leq 2$

3) L'unica soluzione stabile è $(0,0)$ se $0<\lambda<2$

$\lambda=2$ pto di biforcazione instabile

4) $T = \frac{1}{2} ml^2 \left[\frac{1}{3}(1+5\cos^2\theta)\dot\theta^2 + \cos\theta\cos\varphi\,\dot\theta\dot\varphi + \frac{1}{3}\dot\varphi^2 \right]$

$\boxed{13.06.2017}$ ~~(- FILA 1 - CIV & AMB)~~

Es. 1

1) $\mu = \frac{3}{4} m$; $\nu = \frac{1}{4} m$

2) $I_{11} = \frac{4}{3} m l^2$

$I_{22} = \frac{1}{48} m l^2$

$I_{33} = \frac{65}{48} m l^2$

oppure $\underset{\sim}{I}_G = m l^2 \, \mathrm{diag}\left(\frac{4}{3}, \frac{1}{48}, \frac{65}{48}\right)$

3) $I_r = \frac{173}{96} m l^2$

Es. 2

1) $\vec{\omega}_D = - \frac{\dot{\xi}}{R} \vec{k}$

2) $U = -\frac{1}{8} \frac{mg}{R} \left(3\sqrt{3}\, \xi^2 - 2R\xi \right) + c$

3) $\xi_e = \frac{\sqrt{3}}{9} R$ ordinaria sempre stabile $\leftarrow$ 4)

di confine non esistono

5) $\bar{\Phi}_G = \frac{\sqrt{3}}{3} mg \, \bar{i}$

$\bar{\Phi}_H = 2mg \, \bar{j}$ $+ \vec{M} = \frac{2}{9} mgR \, k$

6) $T = \frac{3}{4} m \dot{\xi}^2$

7) $V = -U$

ξ_e è un minimo $\Rightarrow$ centro.

$\boxed{04.07.2017}$ ~~(FILA 1)~~

Es. 1

1) $x_G = y_G = \dfrac{(13+6\pi)}{6(\pi+1)} L$

2) $\underset{\sim}{I}'_H = \dfrac{mL^2}{12(\pi+1)} \operatorname{diag}\left(6\pi+1;\ 6\pi+7;\ 4(3\pi+2)\right)$

Es2

1) $\vec{\omega}_e = -\dot{\theta}\,\vec{k}$

$\vec{\omega}_{D} = -(2\dot{\theta}+\dot{\varphi})\,\vec{k}$

2) $\mathcal{U} = mgR\cos\varphi - 2mg\dfrac{R}{\pi}\theta^2 + mgR\theta + \dfrac{1}{2}mgR\varphi + C$

3) (θ_e, φ_e): $(\pi/4, \pi/6)$; $(\pi/4, \frac{5}{6}\pi)$

$\downarrow$ STAB $\quad \downarrow$ INSTAB $\leftarrow$ 4)

5) $\vec{\Phi}_C = \dfrac{mg}{2}\vec{\imath} + 2mg\,\vec{\jmath}$ esterna

$\vec{\Phi}_H = mg\,\vec{\jmath}$ interna

6) $T = \dfrac{1}{2} m R^2 \left[14\dot{\theta}^2 + 2(1+2\cos\varphi)\dot{\theta}\dot{\varphi} + \dfrac{3}{2}\dot{\varphi}^2\right]$

28.08.2017

Es. 1

1) $I_{11} = I_{22} = \frac{17}{15} m L^2$

$I_{33} = 2 I_{11}$

$I_{12} = -\frac{3}{4} m L^2$

2) $I_r = \frac{113}{60} m L^2$

3) $I_s = \frac{23}{60} m L^2$

4) $I_t = \frac{77}{15} m L^2$

Es. 2

1) $\vec{\omega}_{AB} = \dot{\theta} \vec{k}$

$\vec{\omega}_{AC} = -\dot{\theta} \vec{k}$

2) $U = mgL \left(1 - \frac{\alpha}{2} \cos\theta\right) \cos\theta + c$

3) $\underbrace{\theta = 0, \ \theta = \pi}_{\text{indip. da } \alpha}$, $\theta = \pm\bar{\theta}$ dove $\bar{\theta} = \arccos \frac{1}{\alpha}$ che esistono se $\alpha \geq 1$

4) $\theta = 0$ stabile se $\alpha < 1$

$\theta = \pi$ instabile $\forall \alpha > 0$

$\theta = \pm\bar{\theta}$ stabile se $\alpha > 1$

$\alpha = 1$ pto di biforcazione stabile.

5) per $\theta = 0$ e $\theta = \pi$

$\bar{\Phi}_O (0; \text{indet.})$

$\bar{\Phi}_C$ indet. $\bar{\Phi}_C = \Phi_C \vec{J}$

$\bar{\Phi}_A (0, \text{indet.})$

per $\theta = \pm \bar{\theta}$

$\bar{\Phi}_O = \cancel{\frac{7}{2}}\,3\, mg\, \bar{J}$

$\bar{\Phi}_C = \bar{0}$

$\bar{\Phi}_A = -mg\, \bar{J}$

6) $T = \frac{mL^2}{3} \dot{\theta}^2 (1 + 3\cos^2\theta)$