120. – *Quadro riassuntivo delle funzioni goniometriche di archi particolari.*

Gradi	Radianti	Seno	Coseno	Tangente	Cotangente
0°	0	0	1	0	$\mp\infty$
15°	$\frac{1}{12}\pi$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$2-\sqrt{3}$	$2+\sqrt{3}$
18°	$\frac{1}{10}\pi$	$\frac{1}{4}(\sqrt{5}-1)$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{1}{5}\sqrt{25-10\sqrt{5}}$	$\sqrt{5+2\sqrt{5}}$
$22^\circ 30'$	$\frac{1}{8}\pi$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\sqrt{2}-1$	$\sqrt{2}+1$
30°	$\frac{1}{6}\pi$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
36°	$\frac{1}{5}\pi$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\frac{1}{4}(\sqrt{5}+1)$	$\sqrt{5-2\sqrt{5}}$	$\frac{1}{5}\sqrt{25+10\sqrt{5}}$
45°	$\frac{1}{4}\pi$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
54°	$\frac{3}{10}\pi$	$\frac{1}{4}(\sqrt{5}+1)$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\frac{1}{5}\sqrt{25+10\sqrt{5}}$	$\sqrt{5-2\sqrt{5}}$
60°	$\frac{1}{3}\pi$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$67^\circ 30'$	$\frac{3}{8}\pi$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\sqrt{2}+1$	$\sqrt{2}-1$
72°	$\frac{2}{5}\pi$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{1}{4}(\sqrt{5}-1)$	$\sqrt{5+2\sqrt{5}}$	$\frac{1}{5}\sqrt{25-10\sqrt{5}}$
75°	$\frac{5}{12}\pi$	$\frac{1}{4}(\sqrt{6}+\sqrt{2})$	$\frac{1}{4}(\sqrt{6}-\sqrt{2})$	$2+\sqrt{3}$	$2-\sqrt{3}$
90°	$\frac{1}{2}\pi$	1	0	$\pm\infty$	0
180°	π	0	-1	0	$\mp\infty$
270°	$\frac{3}{2}\pi$	-1	0	$\pm\infty$	0
360°	2π	0	1	0	$\mp\infty$

Relazioni tra le funzioni goniometriche di uno stesso angolo

$$\operatorname{sen}^2\alpha + \cos^2\alpha = 1 \,, \quad \operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} \quad \operatorname{sen}^2\alpha = \frac{\operatorname{tg}^2\alpha}{1+\operatorname{tg}^2\alpha} \,, \quad \cos^2\alpha = \frac{1}{1+\operatorname{tg}^2\alpha}$$

Formule di riduzione al primo ottante

$$\operatorname{sen}\left(\frac{\pi}{2} \pm \alpha\right) = \cos\alpha \,, \quad \cos\left(\frac{\pi}{2} \pm \alpha\right) = \mp \operatorname{sen}\alpha \,, \quad \operatorname{tg}\left(\frac{\pi}{2} \pm \alpha\right) = \mp \operatorname{ctg}\alpha$$

$$\operatorname{sen}(\pi \pm \alpha) = \mp \operatorname{sen}\alpha \,, \quad \cos(\pi \pm \alpha) = -\cos\alpha \,, \quad \operatorname{tg}(\pi \pm \alpha) = \pm \operatorname{tg}\alpha$$

$$\operatorname{sen}\left(\frac{3}{2}\pi \pm \alpha\right) = -\cos\alpha \,, \quad \cos\left(\frac{3}{2}\pi \pm \alpha\right) = \pm \operatorname{sen}\alpha \,, \quad \operatorname{tg}\left(\frac{3}{2}\pi \pm \alpha\right) = \mp \operatorname{ctg}\alpha$$

$$\operatorname{sen}(2\pi \pm \alpha) = \pm\operatorname{sen}\alpha \,, \quad \cos(2\pi \pm \alpha) = \cos\alpha \,, \quad \operatorname{tg}(2\pi \pm \alpha) = \pm\operatorname{tg}\alpha$$

Formule di addizione e sottrazione

$$\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha \cos\beta \pm \operatorname{sen}\beta \cos\alpha$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \operatorname{sen}\alpha \operatorname{sen}\beta$$

$$\operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha \operatorname{tg}\beta}$$

Formule di duplicazione

$$\operatorname{sen} 2\alpha = 2 \operatorname{sen}\alpha \cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\operatorname{sen}^2\alpha$$

$$\operatorname{tg} 2\alpha = \frac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$$

Formule di bisezione

$$\operatorname{sen}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\operatorname{tg}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{1-\cos\alpha}{\operatorname{sen}\alpha} = \frac{\operatorname{sen}\alpha}{1+\cos\alpha}$$